Valenta, A. & Enge, O. (2025). Role-play as an arena for preservice teachers' development of mathematical knowledge?. In M. A. Cabot, S. M. Høisæter & V. Vangsnes (Eds.), *Rehearsing for teaching: Theoretical and empirical approaches to a practice-based teacher education* (pp. 417–441). Fagbokforlaget. DOI: https://doi.org/10.55669/oa600517

17

Role-play as an arena for preservice teachers' development of mathematical knowledge?

Anita Valenta[1] and Ole Enge[1]

[1] Norwegian University of Science and Technology

Abstract: The cycle of exploration and enactment, where preservice teachers and a teacher educator collaboratively plan an instructional activity, rehearse it, implement it in schools and discuss it, has proven beneficial for learning to teach mathematics. In particular, the approach has been shown to provide opportunities for developing mathematical knowledge. However, organising such cycles can be challenging practically. This study investigates preservice teachers' development of mathematical knowledge in a similar but less challenging context: work on role-plays. Here, preservice teachers work in groups without continuous supervision and enact instructional activities in role-plays, assuming the roles of teachers and students. Analysis of the three groups' work shows that while they had opportunities to learn mathematics, the utilisation of these opportunities varied. It seems somewhat arbitrary whether the PSTs discuss the mathematical content during their group work and how they do that. Our findings suggest that new mathematical content needs to be thoroughly worked on before or during work on role-plays and that the teacher educator needs to guide that.

Keywords: mathematical knowledge, preservice teachers' learning, role-plays

Sammendrag: Syklus for utprøving og utforskning der lærerstudenter og en lærerutdanner sammen planlegger en aktivitet, øver, prøver ut med elever og diskuterer det, har vist seg å være nyttig arbeid med matematikklærerstudenter. Spesielt, tilnærmingen har vist seg å gi muligheter for deres utvikling av matematisk kunnskap. Imidlertid kan det være praktisk utfordrende å organisere slike sykluser. Denne studien undersøker lærerstudenters utvikling av matematisk kunnskap i en lignende, men mindre praktisk utfordrende kontekst: arbeid med rollespill. Her jobber lærerstudenter i grupper uten at lærerutdanner er i gruppen hele tiden. I tillegg, aktiviteten prøves ikke ut med elever, men i rollespill, med lærerstudenter i rollene som lærer og elever. Vår analyse av tre gruppenes arbeid viser at selv om de hadde muligheter til å lære matematikk, var det stor variasjon i hvordan de utnyttet disse mulighetene. Det virker noe vilkårlig om lærerstudentene diskuterer det matematiske innholdet under gruppearbeidet og hvordan de vil gjøre det. Resultatene tyder på at nytt matematisk innhold må jobbes grundig med før eller under arbeidet med rollespill, og at lærerutdanneren må veilede det.

Nøkkelord: matematisk kunnskap, læring hos lærerstudenter, rollespill

Introduction

Ball and Cohen (1999) emphasised that teaching must be learned through practice, not just through preparation for practice (p. 12; also see Grossman et al., 2009; Charalambous & Delaney, 2019). This highlights the necessity for practice-based approaches in teacher education.

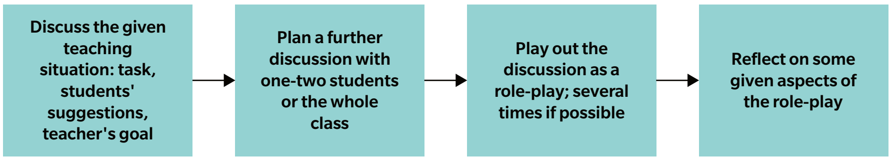
One such approach, which has significantly influenced, and been implemented in, Norwegian teacher education (e.g. Rø et al., 2019; Hovtun et al., 2021), is the cycle of exploration and enactment proposed by Lampert et al. (2010, 2013). In this cycle, a group of preservice teachers (PSTs) and a teacher educator work together to plan an instructional activity. They rehearse the plan before implementing it with a group of students and subsequently discuss the implementation. In addition to learning more general pedagogical aspects of practices, studies show that such joint planning and rehearsals can be an arena where PSTs develop mathematical knowledge and learn to utilise it in practice (Hovtun et al., 2021; Ghousseini, 2017; Rø et al., 2019; Silver et al., 2007).

The role of the teacher educator appears to be significant here. The way teacher educators facilitate discussions and engagement with mathematical content is emphasised in Ghousseini's study (2021), and implicitly in several other studies, too (Lampert et al., 2013; Rø et al., 2019; Hovtun et al., 2021). Hovtun et al. (2021) conjecture that the guidance of teacher educators in the PST groups is one of the crucial factors for providing opportunities for PSTs' development of mathematical knowledge and learning to utilise it in practice during the work on cycles of exploration and enactment.

However, the practical organisation of cycles of exploration and enactment is challenging within teacher education programmes that accommodate a large number of PSTs. Ensuring the presence of a teacher educator in each PST group throughout the whole work cycle and securing sufficient school classrooms for the enactment of planned activities, can be difficult. Therefore, we suggest that other practice-based approaches in mathematics teacher education be tried out, and their potential for PSTs' learning investigated.

Recently, our teacher education programme initiated a trial focusing on role-plays. Our approach diverged from the traditional exploration and enactment cycles, as Lampert et al. (2013) proposed. In the role-play work, PSTs worked in groups without continuous supervision from a teacher educator. Moreover, our role-play work did not incorporate enactments in real school environments. Instead, it revolved around role-plays enacted within the groups. The various stages of the group work on role-plays are depicted in Figure 17.1.

Figure 17.1



The teacher educator designs and presents a teaching situation. This typically involves a mathematical task that an imagined school class is tackling, written solutions or comments provided by a few students in the class, and the teacher's objective for further discussion with some or all the students. The presentation can also involve a brief discussion of the mathematical content in the given teaching situation.

The PSTs then work in groups (comprising 3–5 PSTs) on each of the four stages in Figure 17.1. During the group work, the teacher educator circulates among the groups, supporting the PSTs. Between the stages of the group work, the teacher educator initiates brief whole-class discussions about certain aspects of the work. One of the role-plays may be performed in front of the entire class between the final two stages.

To deepen our understanding of the potential for PSTs' learning within the context of role-plays in mathematics teacher education, we pose the following research question in this study:

> *How can PSTs' group work on role-play serve (or not) as an arena for developing mathematical knowledge and learning to utilise such knowledge in practice?*

To answer the question, we closely examine one of the sessions in which some new mathematical content was included in the role-play. The teacher educator briefly introduced the new mathematical ideas and notions before the PSTs worked on role-plays within groups. We recorded the work of three groups, examining how they approached mathematical ideas, expanded their knowledge, and applied it. The analysis provides more knowledge about role-play's potential for fostering mathematical understanding in teacher education.

Mathematical knowledge for teaching and its development

Lee S. Shulman (1986) identified two main categories of knowledge specifically related to teaching a particular subject: *subject matter knowledge* and *pedagogical content knowledge*. He emphasises that the two aspects are closely linked. Based on Shulman's work, Ball and Bass (2003) introduced the term 'mathematical knowledge for teaching' as an overarching term to describe the competencies required to teach mathematics. In this study, we are primarily interested in subject matter knowledge. Ball et al. (2008) identified Common Content Knowledge and Specialised Content Knowledge as the main components of such knowledge. They define the first as mathematical knowledge that is used not only by teachers, but also by others who work with mathematics. It involves being able to solve a mathematical problem and evaluate whether an answer or notation is right or wrong. This is an essential element in mathematics teacher competence but is also a knowledge many others who work with mathematics have.

Specialised Content Knowledge, on the other hand, is mathematical knowledge that is particularly relevant for mathematics teachers. It involves, for instance, awareness of the advantages and disadvantages of using different representations in explanations and argumentation. In work on division, for example, specialised content knowledge includes knowing about two different models for division: partitive and quotative. In partitive division, the total is divided into a given number of groups, and the question is how much it will be in each group, as in the question '33 metres rope is to be shared equally by six persons. How much does each person get?'. In quotative division, the total is divided into groups of a given size, and the question is how many such groups there are, as in the question '33 metres of rope are to be cut into pieces of 4,50 metres each, how many pieces is it?' (see, e.g. Fosnot & Dolk, 2001, for more about the two division models). Understanding the appropriateness of different models in the context of division is a crucial aspect of Specialised Content Knowledge. For instance, it can be challenging to interpret division by a decimal number using the partitive model.

Both Shulman (1986) and Ball et al. (2008) point out that subject matter knowledge is a crucial base for pedagogical content knowledge. However, studies show that PSTs often overlook the importance of profoundly delving into the mathematical content when planning or analysing teaching (e.g. Enge & Valenta, 2010; Santagata et al., 2007; Star & Strickland, 2008). Such omission can lead to rather general discussions of students and teaching.

Many researchers have claimed that rehearsals are fertile ground for PSTs' learning of subject matter knowledge (e.g. Silver et al., 2007; Hovtun et al., 2021). Silver et al. (2007) showed that collaborative case analysis, lesson planning, and discussions gave teachers opportunities to 'work on and learn mathematical ideas' (Silver et al., p. 261). Similarly, Dışbudak-Kuru & Isıksal-Bostan (2023) emphasise that, through a sequence of initial and revised lesson planning for teaching mathematical connections (i.e. between concepts and procedures), PSTs could make inquiries about their content knowledge, as well as pedagogical knowledge.

In the studies mentioned above, as well as other studies examining learning of content in practice-based teacher education (Ghousseini, 2017; Rø et al., 2019; Lampert et al., 2013), the teacher educator is working together with PSTs in a group, and it seems that this is crucial for PSTs' opportunities to learn. As we point out in the introduction, there is a need to investigate learning potential in PSTs' group work where there is no teacher educator to steer and guide the discussion about and/or rehearsals of teaching. Therefore, we study how PSTs engage with mathematical content knowledge during group work on role-play.

Method

Context and data

The data we analyse in this paper is collected within the Mathematics 1, 1–7 course in Norwegian teacher education for teaching grades 1 to 7. The course is compulsory, has 30 credits, and combines topics within mathematics, its teaching, and learning. At our university, the course is spread over three semesters: PSTs' second, third, and fourth semesters in their five-year teaching program.

Recently, an intervention was conducted in the first part of the course (PSTs' second semester) to investigate work on role-play as a possible approach to work on mathematical reasoning. Several lessons involving role-plays were jointly designed by teacher educators teaching the course and researchers. The starting point of each role-play was an imagined teaching situation, as a task some students in some grade work on (often with some of their solutions or utterances), and a goal the teacher had for her discussion with some of the students or the whole class. The work on role-play was a central element in each lesson. The intention was that PSTs, through such work, could get the opportunity both to develop their mathematical knowledge for teaching and practice using such knowledge in planning, enactment, and reflections of discussions.

In this study, we want to investigate PSTs' engagement with the mathematical content during work on role-play and see whether, and how, it can serve as an arena for PSTs' development of more profound mathematical knowledge. We choose, therefore, to analyse data from one of the designed lessons where the mathematical content involved some aspects within such knowledge that were rather new for the PSTs: different models of, and reasoning within, division. Our experience as teacher educators is that PSTs are unfamiliar with this content from their earlier education.

The lesson started with the teacher educator's short presentation of several moments that were new for PSTs. These included the need to use some models to reason about division, the existence of two models (often, students and PSTs think only about partitive division as division), and how they can be used. Examples of the two models were given: 238 kroner are to be divided between 15 persons – how much money to each, as a partitive model, and 5 metres of material to be cut in 0.5 metres to make scarves – how many scarves, as a quotative model. As teacher educators, we know that PSTs need to work on such aspects in different ways, be challenged to design contexts, make drawings, and reason. Here, the hope was that this would happen during the work on the following role-play task:

Part 1

Which gives the greatest answer? Find out **without calculating**.

a. 24:12 or 24:4
b. 18:4 or 18:10
c. 5:1 or 5:0.5
d. 3:4 or 3:7
e. 238:15 or 238:16
f. 7:3 or 7:0.3
g. 12:0,1 or 12:0.01
h. 2.4:1.1 or 2.4:0.11
i. 21:0.3 or 21:0.000000001

Part 2

In a grade 6 class, students have worked in pairs with this activity. The teacher's goal for the lesson is that students discover, through the exploration of these tasks, a correlation between the divisor and the quotient (when the dividend is unchanged).

Two students have done the calculation but could not find the correlation. The teacher comes to their table.

- Plan a discussion in which the teacher elicits, responds to and promotes the students' reasoning.
- Allocate roles in the group and conduct the discussion: two are students, one is a teacher, the fourth is an observer.

Part 3

Reflection based on the role-play in the group and in the whole class:

- What processes within mathematical reasoning come into play, and when?
- What teaching moves are used, and how did they support students' mathematical reasoning?

PSTs worked in groups on the task, but there were whole-class discussions during the work. During work on part 2, the teacher educator pointed out that the observer should make notes during the role-play, and that the group should discuss the teacher's questions and possible responses before trying out the role-play again and refining it. To end part 2, one group of PSTs enacted their role-play for the whole class.

The data we analyse in this study are the transcriptions of the three PST groups' work on all three parts above (it was video recorded). In other words, we analyse all the stages of their group work on role-play (Figure 17.1). This includes their discussion of the task imagined students work on, their planning of discussion between a teacher and two of the students, their enactment of the planned discussion as a role-play, and their reflections on their own role-play and the one which has been enacted in front of the whole class.

Method of analysis

In this study, we are interested in how PSTs work with mathematical content, discuss mathematical problems and ideas, and develop and use their mathematical knowledge. We conducted a thematic analysis (Braun & Clarke, 2006) of the transcripts. The first step was identifying excerpts in which PSTs discussed mathematical content.

The next step of the analysis was deductive, led by the notions of Common Content Knowledge and Special Content Knowledge, and what they entail for the case of reasoning on division: In each excerpt where mathematical content was discussed, we noted which division problems the PSTs talked about, what questions they discussed, how they proceeded to find the answers, which model (partitive or quantitative, if any) they used and how – which context (money, scarves, cakes) and how (clarity of the context and questions).

In the end, we made an overview of the different excerpts for each group to get a complete picture of how the mathematical content was treated throughout the group work.

Analysis

We present our findings for the three PST groups' work.

Group 1

The group consists of Mia, Siri, Marie, and Emilie. Ole and Oda are also in this group to begin with, but they join group 2 when the discussion is to be enacted in role-play (the teacher educator asks them to do that, so the number of PSTs in each group is appropriate).

After working on the task for just a few moments, Oda states the conjecture in rather technical language, 'If the dividends are equal, then the one with the lowest divisor is the biggest.' Not all group members seem sure about this conjecture, since they continue to search for one.

> *43. Siri: Okey, if you divide five by one and by 0.5. Maybe we need to show that five divided by one is bigger even if 0.5 is less?*
>
> *44. Mia: Yes.*
>
> *45. Marie: Okey, but if you have a whole person, then divide five on the whole person? Versus if you have half a person.*
>
> *46. Siri: You still have just five.*
>
> *47. Marie: I don't' know. This was very hard.*
>
> *48. Siri: It does not make any sense.*

Siri raises the question of comparing five divided by one and five divided by 0.5. In turn 45, Marie introduces a situation indicating partitive division, which they struggled to make sense of when dividing by decimal numbers. They do not manage to resolve this problem, and the discussion continues.

> *78. Siri: Five divided by one will be five. And five divided by 0.5?*

79. Ole: It will be 10.

80. Siri: Are you sure?

81. Marie: This was my thinking, that you will get twice as much.

82. Ole: It depends on what the task asks for. If you sell games, then you cannot divide the game two times in a way. So, then it will not be a valid answer.

83. Marie: If you have five, you do not suddenly get ten.

84. Ole: Yes.

85. Marie: Then you must divide the five into two.

86. Siri: Okey, we have five cakes.

87. Marie: Then we divide them in two [uses the calculator]. It is ten, yes.

Both Siri and Marie are unsure about division by decimal numbers. Marie does not understand how one can get ten starting with five, indicating that she thinks about division as partitive division. They do not discuss this issue about division further. Ole and Oda leave for another group before the role-play.

Siri volunteers to be the teacher and jokes that neither the PSTs playing students nor she understands what the tasks ask for. The role-play starts with whole number divisions, where Siri (the teacher) introduces a context of muffins and persons to reason about the tasks 24:12 and 24:4. The PSTs playing students correctly reason that the fewer persons, the more muffins each person will get. Then Siri moves on to five divided by one and five by 0.5.

158. Emilie: If you have five cakes and divide by one, you will get five cakes. If you have five cakes and divide by 0.5… I do not know.

159. Siri: If you have five cakes and divide them in halves, you just divide each cake in half. How many cakes do you get?

160. Emilie: Ten halves.

161. Siri: Ten halves that is correct. Very good.

None of the PSTs grasp here that the context has changed from dividing cakes between persons to halving cakes, and that this change will not help them to reason about the task. They continue with their role-play and form the conjecture that 'the smaller the number you divide by, the bigger answer you get.' Generally, they are very pleased with the role-play, and Siri states, 'This was really good. I feel that we have done everything now.' When discussing the next, refined role-play, Mia, who has taken notes of the role-play, sums up:

206. Mia: Then there were some cakes. Five cakes divided by one is five, and five divided by 0.5 is ten half-cakes. No, ten cakes.

207. Marie: Yes, this did not make that much sense.

208. Mia: It will be halves.

209. Emilie: No, but then it will not be a whole.

210. Mia: It will be ten, but they will be smaller.

212. Marie: It is so funny that if you are to divide by half a person, then you have to divide the cake in two. The half-person can get.

213. Siri: It just is not a half-person. It is more like five divided by half.

214. Marie: Oh, so you do not divide persons?

215. Siri: You only divide the cake.

> *216. Marie: The person that is half, that is the person getting the cake? No, I do not know. Yes, but… just continue.*

In the excerpt above, Marie is persistent in her quest to understand what happens when one divides by decimal numbers, but the others do not offer much help, and finally, she gives up.

The talk about the division of decimal numbers (5:1 versus 5:0.5) in the refined role-play is similar to the previous play. The teacher asks the students how many pieces of cake one gets if one divides five cakes in half. After a student says ten, the teacher concludes that this is correct and more than five divided by one. Together, they state the conjecture that 'if you have the same dividend but different divisors, then the one with the smallest divisor gives the biggest answer.' Siri (the teacher) asks the students to check the conjecture on different tasks, and then they end the role-play. The talk after the role-play is mainly about being a teacher in general, and they do not discuss the play or talk about mathematics or teaching mathematics.

Another group (group 2) then performed their role-play in front of the class. Here, they discussed 12 divided by 0.5 using a quantitative context (scarves, 1 metre long or 0.5 metre long). The PSTs in group 1 are impressed by the other group's role-play, particularly the teacher's moves.

> *344. Siri: He expands on the contribution of the students. Giving alternative solution strategies, using that one with the scarf. It was different than we thought about.*

> *345. Mia: That was smart.*

They do notice the scarf context, but do not elaborate on why it was 'smart', or how to use it when discussing division by decimal numbers.

Despite Maria repeatedly questioning the reasoning behind using half cakes and half persons during group work, the PSTs in group 1 failed to resolve the issue. They exclusively relied on a partitive model and discussed distributing cakes among persons.

Even though the teacher educator introduced one quotative context (involving scarves) suitable for division by decimal numbers before the group work, group 1 never discussed it during their work. When the other group uses it in their role-play, they do not go into what was good with it compared with the model they have used (half cakes/people).

By not resolving the issue of half cakes and half persons, and not discussing another context for division, we suggest that the role-play did not play a productive role in developing mathematical knowledge for these PSTs.

Group 2

To begin with, only Mona and Tora are in the group. Ole and Oda are in group 1 during work on part 1 of the task; they join group 2 for parts 2 and 3.

Mona and Tora start the work by considering 18:4 and 18:10. Tora suggests that '18:4' is the biggest, and they go on to compare 5:1 and 5:0.5, as shown in the excerpt below:

> *4. Tora: Five divided by one is biggest because… hmm.*
>
> *5. Mona: Isn´t it? Or?*
>
> *6. Tora: I don't know. If you divide five by 0.5. You divide by one half. Is it more then?*
>
> *7. Mona: I don´t know.*
>
> *8. Tora: If you multiply by 0.5, it is 2.5.*
>
> *9. Mona: I have to use the calculator.*

Mona and Tora continue to use the calculator (even though the task asks not to do that), and they conclude that 'it seems that, as long the first number is the same, a smaller second number gives a bigger answer.' Here, Mona and Tora show they are very insecure about the mathematical content. They do not reason about the relation between 18:4 and 18:10; no possible explanation is given. It seems they have no other way to reason about the division with 0.5 than to check on the calculator. Mona explicitly expresses her struggle with the mathematical content when Oda and Ole join the group for the role-play:

'I can be a student; I need to have it explained.' Tora will also be a student. Ole gets the role of teacher, and Oda is an observer.

The teacher Ole starts by asking the students to compare 200:4 and 200:5. He leads to reasoning using a partitive context, sharing money between four or five persons and 'we need to share between more people, then they get less', before going further to comparing 12:1 and 12:0.5. The student Tora starts to use the same money context, but the teacher Ole suggests:

> *95. Ole: It is difficult to share by a half-person. Let us try another context: Say you have 12 metres of scarf, and you are to divide it in 1 metre or divide it in 0.5 metres. How many scarves do you get?*
>
> *96. Mona: You get one whole scarf if you are dividing by 1, so you get a piece.*
>
> *97. Ole: Mhm, if you want one piece, yes (laughing). It was a bad context. Timeout, I need to re-formulate. Help me, Oda. You have 12 metres of scarf which you are to divide into 1 metre, so you get 12 pieces, right?*
>
> *98. Oda: Yes.*
>
> *99. Ole: And if you are to divide it into 0.5 metres, you get 24. Okey?*
>
> *100. Oda: Yes, you do.*
>
> *101 Ole: Right, timeout is finished. I will reformulate, girls. We have a 12-metre-long scarf that is to be divided into pieces that are 1 metre long. And then you have a scarf, which is also 12 metres long and has to be divided into pieces that are 0.5 metres long. Which one gives more pieces?*

Tora and Mona suggest that it is the last one, and they also reason further by saying that the number of scarves will be doubled because '50 centimetres plus 50 centimetres is one metre.'

Teacher Ole continues by asking the students to look for some connection between the two comparisons, and Mona suggests:

> *107. Mona: If there are fewer people to share the money, they get more. But it is the same with scarves, too: if you divide them into more pieces, there is more. So, that can be a connection here.*

The group finishes the role-play here. Mona says it was lovely to be a student and that they 'figured it out in the end.' There is development in Mona and Tora's mathematical knowledge, as we see it: unlike their work on the task before the role-play, they use a partitive context, and develop arguments (without calculating) for why 200:4 must be more than 200:5, and they use a quotative context to reason about the relation between 12:1 and 12:0.5.

Still, Mona's utterance in turn 107 shows that she still struggles with quotative division as she talks about quotative as 'dividing in more pieces,' instead of dividing into pieces of a given size.

Also, Ole seems to have used the work on the role-play to develop his mathematical knowledge. He tries to suggest a quantitative context that the teacher educator mentioned at the beginning of the lesson, but his language is imprecise. He notices that and takes a time-out to discuss it with Oda before moving further.

After the role-play, the group points out the significance of the teacher's preparation: planning how to explain, selecting examples, and using appropriate contexts. For instance, the context of a scarf was effectively used to reason about division by 0.5. Mona asks whether some context with cakes would work for 12:0.5.

> *186. Mona: If you have a whole cake that you divide…*
>
> *187. Tora: You need to deliver it in 12 birthday parties.*
>
> *188. Ole: Yes, you have a cake and divide it in pieces that are half of cake-pieces. How many do you get?*
>
> *189. Mona: If you first divide it in 12, and then each piece in halves. Something like that.*

190. Ole: Yes, you have a cake with 1-pieces, and divide pieces in halves. How many do you have?

191. Oda: The example with the scarf was good.

192. Tora: If one rather says that we are to cut, to make smaller scarves?

193. Ole: To make it simpler?

194. Tora: Yes. Because I could not understand it to start with, I just thought, why do we have such a long scarf? Why should we divide it? It can be just me being too slow. But maybe if you said that we had scissors and needed to cut.

The group tries to find how such contexts can match 12:0.5 and discuss how to improve the scarf context. They try the role-play again, this time using cakes as the context. Again, Ole is the teacher, Mona and Tora are students, and Oda is the observer.

237. Ole: So, we have a cake. And we divide it into 12 pieces. And in this one, you divide all the 12 pieces into… no. (laugh) Time out.

238. Oda: There is something wrong.

239. Ole: No.

240. Mona: I don't understand why divide in 12, because…

241. Tora: Because you have 12 cakes. No. You have one cake. (laugh)

242. Ole: Quiet, quiet (laugh).

243. Mona: No, but I am a student now, really. Here, you have 200 to divide into four pieces, or between four persons. Here, you have 12 of something to divide into one piece.

244. Ole: If you have a cake with 12 pieces, and you are to find how many pieces of full size, size 1 (laugh). I agree; it does not work.

245. Tora: We go back to the scarf.

246. Ole: Yes, I agree.

The group continues the discussion in the same way as in the first role-play – the same question as in turn 101, reasoning about 50 cm and 1 metre, and conclusion as in turn 107. The teacher educator asks them to perform their role-play in front of the class, and they do that. After that, in their reflection in the group, Oda brings up the notions of partitive and quotative division and points out that they were important in the task.

As shown in the excerpts above, PSTs in group 2 discuss the mathematical content throughout the work. In their first discussion on the task, Mona and Tora do not use any context, and do not manage to reason about the division problems. With Oda's help, Ola brings in a quotative context (scarves) during the role-play – in a similar way as the teacher educator presented it in the beginning. Ole's question is partly imprecise, and the conclusion – as expressed in turn 107 – is also imprecise. However, the PSTs go into further discussion and try to design a quotative context with cakes too. They fail, but the discussion, and their new try-out, are highly valuable as they try to make sense of the context, match it with 12:0.5, and find out why their attempts do not make sense by comparing the structure with other division contexts (as in turn 243).

We suggest that the work on role-play serves as a productive arena for developing the mathematical knowledge of these PSTs. This involves learning about and investigating various division contexts, understanding their applicability, and effectively utilising this knowledge. As they experiment with different scenarios in the role-plays, they explore how to formulate clear questions, and how the conclusion can be developed and expressed.

Group 3

The group consists of Marius, Ane, and Gry. They start the work by concluding immediately that more will be given to each person when the divisor is smallest.

> *2. Marius: But, is it not always the smallest factor when the first number is the same?*
>
> *3. Ane: Yes, it must be like that, I guess.*
>
> *4. Marius: Yes.*
>
> *5. Ane: So, it will be more for each person.*

Later, when they discuss the conjecture, they are more precise and state, 'When the divisor is smallest, the answer will be biggest'. When it comes to the relation students can find, they formulate it as 'the smaller the divisor, the bigger the answer.' PSTs in this group do not refer to any of the examples given in the task, and the only reference to division involving decimal numbers is Marius mentioning, 'I think that example is so funny. That first you divide by a whole person and then by a half person.'

In the role-play, Gry is the observer, Ane plays the student, and Marius is the teacher. The role-play starts with Marius asking, 'What have you found out?' whereupon Ane answers, 'I found out that those with the smallest divisor give the biggest answer.' They take a timeout and realise that they were 'too fast' and that Ane was 'too clever and stated the conjecture too early.' They discuss several further examples, like 3:4, 3:7, 200:40, and 200:10. However, they never talk about division by decimal numbers in the time-outs nor the role-play. The only model they use when deciding which task gives the biggest (or smallest answer) is the partitive model (like 'the more persons you have to divide by, the less you get').

After the first role-play, they discuss various pedagogical moves to emphasise students' thinking and generalisation, but do not discuss division by decimal numbers. Still, in the refined role-play, Marius (playing teacher) asks about one task involving decimal numbers.

184. Marius: Yes…what have you found out in task a?

185. Ane: In a, I found out that it will be the biggest answer for 24:4.

186. Marius: Yes, the biggest answer is 24:4. What about in task h?

187. Ane: In h, it will be 2.4:0.11.

188. Marius: Do you see any connection between the tasks? Something that is the same?

189. Ane: I can see that when the divisor is smallest then the answer will be biggest, maybe?

190. Marius: Yes, but why?

191. Ane: Well, maybe because you divide something between fewer people then there will be more on each.

192. Marius: Yes, that's correct.

There is no explicit discussion about why 2.4:0.11 is the biggest (compared with 2.4:1.1), and when arguing for why the smallest divisor gives the biggest answer, they use a context of people, indicating a partitive model of division. The teacher educator comes by, and she asks them to play their role-play once more. Again, the PSTs use a partitive division model in their argumentation. The teacher educator notices this and asks them if the teacher (in the role-play) could have offered another context making sense in the case of decimal numbers.

255. Marius: Maybe we could have used quotative division?

256. Teacher educator: Yes. How would that be? Discuss it, how it can sound.

The teacher educator goes to another group.

257. Gry: That what she said before with the scarf?

258. Marius: I don´t remember completely.

259. Gry: If you have five a five-meter-long scarf and are to divide it into 1-meter pieces, as in task c. You get five pieces. If you have five meters and are to divide it into half-meter pieces, you get ten pieces.

260. Ane: Yes, with these numbers, but how could we do it with 2.4:0.11?

261. Gry: It does not make sense.

262. Marius: Doesn´t it?

263. Gry: Or, you can have 2.4 meters of rope, and you want each piece to be 0.11. How many pieces do you get?

264. Ane: Yes.

As we see above, when the teacher educator asks about another, more appropriate context, the group answers immediately that they could use quotative division. When trying to formulate the context, they think about the one the teacher educator used, and they manage to use it in a precise way for five divided by one/0.5 (turn 259). Even though they are a bit unsure whether and how it can be used with other numbers, they succeed in formulating another quotative context. Afterwards, group 2 presents their role-play in front of the class. In their reflection afterwards, group 3 discusses only pedagogical aspects of the role-play, and does not discuss models and reasoning in the division.

Thus, when the teacher educator asks directly about context that would make sense in division by decimal numbers, the PSTs in this group show that they know the notion of quotative division (turn 255). They understand the context the teacher educator used as an example (turn 259), and they can formulate such context in other examples (turn 263). However, this is the only time they mention it, and they do not discuss how such a context could be used in argumentation of division tasks.

Thus, the data indicate that the PSTs in this group can use this quotative context to give meaning to division, including decimal numbers. However, it does not seem that they find it important to do in the role-play, and they do not utilise their knowledge without the teacher educator's explicit intervention.

Discussion and implications

This study aimed to investigate how role-plays can contribute, or not contribute, to the development of PSTs' mathematical knowledge when they worked in groups without continuous guidance from teacher educators. Our findings indicate that each of the three groups studied had opportunities to learn mathematics, but it varied in how effectively they utilised these opportunities.

The findings show that PSTs in both Group 1 and Group 2 engage in the mathematical content without the teacher educator's intervention. In Group 1, the discussion about the context suitable for division by decimal numbers is primarily initiated by one of the PSTs, Marie. She reacts to the context proposed by the group, and highlights aspects that do not align with the division problem. The other PSTs try to fill the gaps, but it does not work (the context does not make sense, as Marie points out); Marie gives up, and the group does not go further.

However, they notice a more appropriate context used by another PST group later; they think it is a 'clever move' and talk about how this is used to 'bring out the reasoning of the students'. So, even though they do not comment on this other context in depth, they may understand better what an appropriate context for work on decimal numbers can be. Thus, the PSTs in the group initiated and took the discussion on mathematical content. Still, it is unclear whether the role-plays have served as an arena for the development of mathematical knowledge.

On the other hand, PSTs in Group 2 try to use the division context suggested by the teacher educator at the beginning of the lesson in the role-play. They manage to use it in the role-play (to some extent), and then go into further discussion about alternative contexts, which they try out in the next role-play, and discuss further.

Even though some aspects still seem challenging for the group, we suggest that the PSTs develop their mathematical knowledge and try to utilise it in the role-plays. Dışbudak-Kuru and Isıksal-Bostan (2023) point out in their study that PSTs can make inquiries about content knowledge during lesson planning, and we see that both Groups 1 and 2 do it in our study. However, Group 1 stops the inquiries rather early, while Group 2 manages to go further.

In Group 3, the PSTs neither discuss nor use any context in the role-plays that could be appropriate for division by decimal numbers. When the teacher educator asks them about this, they immediately mention the notion of quantitative division, give an example of the context, and also, without significant challenges, design another similar context that makes more sense for smaller numbers.

Thus, it seems that they do not really have problems with the mathematical content, but they simply do not use their knowledge in the role-play. Studies have shown before that PSTs overlook the importance of going into depth of the mathematical content (see, e.g., Enge & Valenta, 2010; Santagata et al., 2007; Star & Strickland, 2008). We see that this is the case both in Group 3 (not discussing it at all) and in Group 1 (giving up without resolving the problem).

Joint work on planning and rehearsing discussions has been shown earlier to give opportunities for PSTs' learning of subject matter knowledge (e.g., Silver et al., 2007; Hovtun et al., 2021). In these studies, a teacher educator works together with a group of PSTs. Hovtun et al. (2021) propose that the teacher educator's guidance is an essential aspect in this context.

Our findings corroborate their assertion. In groups consisting of just PSTs, it seems somewhat arbitrary whether the PSTs will discuss the mathematical content during their group work and how they will do that. Furthermore, when they do not discuss it sufficiently, their focus shifts to rather general considerations about students and teaching, and their learning opportunities are reduced.

The development of mathematical knowledge through work on role-plays, where PSTs work in groups without a teacher educator present the whole time, seems to be more challenging to achieve than in usual cycles of investigation and enactment (Lampert et al., 2013).

Still, working on role-play is a valuable approach in teacher education, but it is essential to be aware of this approach's advantages and limitations.

Our findings suggest that new mathematical content needs to be thoroughly worked on before or during the PSTs' engagement in role-plays, and that the teacher educator needs to guide this.

References

Ball, D. L. & Bass, H. (2003). Toward a practice-based theory of mathematical knowledge for teaching. In E. Simmt & B. Davis (Eds.), *Proceedings of the 2002 annual meeting of the Canadian Mathematics Education Study Group* (pp. 3–14). CMESG/GCEDM.

Ball, D. L. & Cohen, D. K. (1999). Developing practice, developing practitioners: Toward a practice-based theory of professional education. In G. Sykes & L. Darling-Hammond (Eds.), *Teaching as the learning profession: Handbook of policy and practice* (pp. 3–32). Jossey-Bass.

Ball, L. D., Thames, M. H. & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education*, *59*(5), 389–407.

Braun, V. & Clarke, V. (2006). Using thematic analysis in psychology. *Qualitative Research in Psychology*, *3*(2), 77–101.

Charalambous, C. Y. & Delaney, S. (2019). Mathematics teaching practices and practice-based pedagogies: A critical review of the literature since 2000. *International Handbook of Mathematics Teacher Education, 1*, 355–390.

Dışbudak-Kuru, Ö. & Işıksal-Bostan, M. (2023). Supporting the development of preservice teachers' mathematical connection skills in a teacher education programme. *International Journal of Mathematical Education in Science and Technology*, *54*(8), 1393–1419.

Enge, O. & Valenta, A. (2010). Utvikling av matematikklærerkompetansen hos studenter i allmennlærerutdanning. *Tidsskriftet FoU i Praksis*, *4*(3), 61–77.

Fosnot, C. T. & Dolk, M. (2001). *Young mathematicians at work: Constructing multiplication and division*. Heinemann.

Ghousseini, H. (2017). Rehearsals of teaching and opportunities to learn mathematical knowledge for teaching. *Cognition and Instruction*, *35*(3), 188–211.

Ghousseini, H. N. (2021). The work of coaching in rehearsals to enlist mathematical knowledge for teaching. In *Developing mathematical proficiency for elementary instruction* (pp. 165–181). Springer International Publishing.

Grossman, P., Hammerness, K. & McDonald, M. (2009). Redefining teaching, re-imagining teacher education. *Teachers and teaching: Theory and Practice*, *15*(2), 273–289.

Hovtun, G., Mosvold, R., Bjuland, R., Fauskanger, J., Gjære, Å. L., Jakobsen, A. & Kristensen, M. S. (2021). Exploring opportunities to learn mathematics in practice-based teacher education: A Norwegian case study. In T. Bergqvist, K. Hemmi, & T. Meaney (Eds.), *Bringing Nordic mathematics education into the future: Preceedings of Norma 20: The ninth Nordic conference on mathematics education Oslo, 2021*. SMDF, Swedish Society for Research in Mathematics Education Göteborg.

Lampert, M., Beasley, H., Ghousseini, H., Kazemi, E. & Franke, M. (2010). Using designed instructional activities to enable novices to manage ambitious mathematics teaching. In M. K. Stein & L. Kucan (Eds.), *Instructional Explanations in the Disciplines* (pp. 129–141). Springer.

Lampert, M., Franke, M. L., Kazemi, E., Ghousseini, H., Turrou, A. C., Beasley, H., Cunard, A. & Crowe, K. (2013). Keeping it complex: Using rehearsals to support novice teacher learning of ambitious teaching. *Journal of Teacher Education, 64*(3), 226–243.

Rø, K., Valenta, A., Langfeldt, M. B. & Ødegaard, R. P. (2019). Joint academic planning. *Tangenten – Journal of Mathematics Education*, *2019*(4), 40–52.

Santagata, R., Zannoni, C. & Stiegler, J. (2007). The role of lesson analysis in preservice teacher education. *Journal of Mathematics Teacher Education, 10*(2), 123–140.

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher, 15*(2), 4–14.

Silver, E. A., Clark, L. M., Ghousseini, H. N., Charalambous, C. Y. & Sealy, J. T. (2007). Where is the mathematics? Examining teachers' mathematical learning opportunities in practice-based professional learning tasks. *Journal of Mathematics Teacher Education, 10*, 261–277.

Star, J. & Strickland, S. (2008). Learning to observe: Using video to improve preservice mathematics teachers' ability to notice. *Journal of Mathematics Teacher Education, 11*(2), 107–125.