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Preservice teachers' role-plays of discussions promoting students' mathematical reasoning

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Abstract: Learning to lead discussions that promote students' mathematical reasoning is challenging for preservice teachers. In such discussions, the teacher needs to involve students and help them to express and extend their reasoning. At the same time, the teacher needs to promote the building of mathematically accurate arguments. To support preservice teachers' learning, we used role-plays of discussion where a teacher was to support some students' reasoning. The discussions were to be planned by groups of preservice teachers, and they played all the roles. We observed different challenges during planning: superficiality, too demanding mathematical complexity and unequal participation. Here, we analyse what characterises teachers' communication moves and the structure of arguments developed in the enacted role-plays in three groups that displayed challenges during planning. Our findings indicate that the planning challenges clearly impacted both the teachers' moves and the structure of arguments in the enactments of the role-plays. The results indicate a need to find a way to strengthen the planning phase, particularly when working with complex mathematical content as reasoning.

Keywords: mathematical reasoning, role-plays, teacher education, mathematical discussions

Sammendrag: Å lære å lede diskusjoner som fremmer elevenes matematiske resonnering er utfordrende for lærerstudenter. I slike diskusjoner må læreren involvere elevene og hjelpe dem til å uttrykke og utvide resonnering. Samtidig må læreren fremme oppbyggingen av matematisk korrekte argumenter. For å støtte lærerstudenters læring brukte vi rollespill av diskusjoner der en lærer skulle støtte noen elevers resonnering. Diskusjonene skulle planlegges i grupper, og lærerstudentene spilte alle rollene. Vi observerte ulike utfordringer under planleggingen: overfladiskhet, for krevende matematisk innhold og ulik deltakelse. Her analyserer vi hva som kjennetegner lærernes kommunikasjonsgrep og strukturen til argumenter utviklet i de gjennomførte rollespillene i tre grupper som viste utfordringer under planleggingen. Våre funn indikerer at planleggingsutfordringene tydelig påvirket både lærernes kommunikasjonsgrep og strukturen til argumentene. Resultatene viser et behov for å finne en måte å styrke planleggingsfasen på, spesielt når man jobber med komplekst matematisk innhold som resonnering.

Nøkkelord: matematisk resonnering, rollespill, lærerutdanning, matematiske diskusjoner

Introduction

Mathematical discussions are crucial for students' learning, and when orchestrating them, teachers should actively build on students' reasoning and allow students to engage in conjecturing and justification (Shaughnessy et al., 2019). Hence, mathematical discussions involve positioning students as sense-makers when working on mathematically rigorous content. The teacher must ask questions that elicit and promote the students' thinking, which is contrary to the teacher explaining and doing all the mathematical work (Drageset, 2014).

Moreover, allowing students to engage in conjecturing and justifying entails emphasising mathematical reasoning (MR), which involves processes for developing and validating mathematical statements (Jeannotte & Kieran, 2017). Emphasising MR in school mathematics is essential to provide students with opportunities to learn what mathematics is about, and for deep learning of mathematics (e.g. Ball & Bass, 2003). However, MR, particularly developing mathematically valid arguments, is a known challenge for students, teachers, and preservice teachers (PSTs) (Stylianides et al., 2017). Consequently, leading mathematical discussions to support students' MR is challenging, due to the improvisational work of teaching (Lampert et al., 2013) and the complexity of the mathematical content.

Previous studies on orchestrating mathematical discussions show that teachers and PSTs tend to use 'low potential' moves, such as simplifying tasks, focusing on facts, procedures, or asking closed questions (Drageset, 2014; Ellis et al., 2019; Skott & Valenta, 2022). However, by taking a practice-based approach to teacher education, and implementing learning cycles (Lampert et al., 2013; McDonald et al., 2013), PSTs can learn the practice of leading discussions in a less complex setting than authentic classrooms. In the learning cycle, PSTs investigate, plan, and enact a mathematical discussion before reflecting on the enactment (McDonald et al., 2013).

Studies have investigated, and broadly positively reported on, the implementation of learning cycles in teacher education (e.g. Lampert et al., 2013; Wæge & Fauskanger, 2023). Still, of the few studies that explicitly focus on MR, the overall conclusion is that learning to support students' mathematical reasoning remains challenging (e.g. Buchbinder & McCrone, 2020). Buchbinder and McCrone (2020) particularly point to PSTs' challenges in using precise mathematical language and predicting students' strategies.

To support PSTs in learning to lead mathematical discussions emphasising students' MR, we designed lessons in which we implemented learning cycles where the PSTs planned and enacted role-plays of mathematical discussions. Role-plays are here understood as simulations of authentic practice, in which PSTs take on roles as teachers and students in a hypothetical scenario (Buldu, 2022; Stürmer et al., 2024).

We investigate the enactments of the role-plays in terms of the communicational moves the PSTs enacted to position students as sense-makers, in addition to the structure of the mathematical arguments developing during the discussions. The study builds on observations made during the PSTs' planning phase, where we noticed various challenges emerging. We investigate the enactments of the role-plays in three different groups, based on three observed planning challenges: superficiality, too demanding mathematical complexity, and unequal contributions by the PSTs in the group. The research question is:

> *What characterises role-plays of discussions promoting MR in terms of teachers' communication moves and the structure of arguments developed, for three groups of PSTs who displayed challenges during planning?*

Planning and enactment in learning cycles in teacher education

In a practice-based approach to teacher education, the curriculum centres around doing practice instead of learning about practice (Ball & Cohen, 1999). An example of doing is to enact certain aspects of practice through approximations of practice (Grossman et al., 2009). Different approximations of practice offer different support and complexity for the PSTs, allowing the enactment of different aspects of practice.

In the learning cycle (McDonald et al., 2013), practice can be enacted in school, in rehearsals, role-plays, or animations, to mention some examples. Rehearsals, role-plays, and animations are less complex than enactments in school in terms of authenticity. However, they allow for focusing on aspects of practice that can be overshadowed by aspects like classroom management (Grossman et al., 2009). Thus, O'Flaherty et al. (2024) argue for approximation in the context of teacher education programmes to provide opportunities for experimenting and learning from mistakes in the safety of the teacher education context.

In rehearsals, a teacher educator closely guides a group of PSTs using time outs as they enact a mathematical discussion (e.g. Lampert et al., 2013), while in animations, practice is enacted through comics, often in software programmes (Amador, 2017). Role-plays, being the focus of this study, differ from rehearsals in terms of less support from the teacher educator, and from animations, in terms of being enacted in real time.

The literature provides evidence of different scenarios of how the relation between planning and enacting discussions plays out. For example, Mendes et al. (2022) found that PSTs faced challenges during planning and enacting discussions to promote students' mathematical reasoning, partly because they struggled to solve the task themselves. In contrast, Buchbinder and McCrone (2020) found that PSTs successfully planned for mathematical reasoning in their lesson but faced challenges implementing the plans.

Regarding the relation between predicted student responses and enacted teacher moves, Hallman-Thrasher (2017) showed that PSTs tended to lower the cognitive demand of their questioning when they responded to the anticipated student contributions, while Orr and Bieda (2023) found that PSTs lowered the academic rigour of their questioning when they faced unexpected situations.

Studying the moves that PSTs in our study planned and enacted, Ødegaard (2023) found that the PSTs planned and enacted several moves related to MR, and that the proportion of MR moves increased from the plans to the enactments. Hence, the literature shows differing results regarding relations between planning and enactment. This study further elaborates on various challenges PSTs face during planning, and how their enactments reflect these challenges.

Theoretical framework

We analyse two aspects of the discussions played out in role-plays: 1) the structure of the mathematical argument constructed through discussions, and

2) the acted teachers' use of communication moves for supporting students' MR. The two aspects are central to discussions aiming to support students' MR (Ellis et al., 2019; Yackel, 2002). Attending to both aspects gives insight into opportunities and challenges associated with cycles of investigation and enactment in mathematics teacher education.

Before presenting the frameworks used to analyse the structure of arguments and the communication moves to support MR, we provide central notions of MR.

Mathematical reasoning

We conceptualise mathematical reasoning as processes for developing or validating statements about mathematical objects and their properties (Jeannotte & Kieran, 2017). Developing statements involves identifying patterns, looking for similarities and differences, classifying, generalising, and conjecturing.

For example, third-grade students can explore the number sequence 8, 16, 24, 32, 40,…. By comparing the numbers, they can observe the pattern that all the given numbers are even. This observation can be extended to a general conjecture that 'all multiples of eight are even.' However, it is not evident in third grade that this statement is generally true (for all multiples of eight), and an argument is needed to show that.

Other types of statements can also require argumentation, for example, conjectures about single cases (e.g. '103 is a prime number') and finitely many cases (e.g. 'all primes below 20 have a twin prime') (Stylianides & Ball, 2008).

There are different arguments for the validity of mathematical statements, ranging from (informal) justifying via proving to formal proving (Jeannotte & Kieran, 2017). Proof is a mathematically correct argument, and in school mathematics, it is a deductive argument that refers to statements and techniques that are known and appropriate to the given community (Stylianides, 2007). Hence, in primary school, proofs can be built on concrete examples and visual representations instead of algebraic notation.

For example, proof of the validity of 'all multiples of eight are even' can be based on using $3 \cdot 8$ as an example. One can argue that since each 8 can be divided into two, the multiple 3 · 8 can be divided into two. In this example, it did not matter that it was three eights – we could have done the same with any number of eights. Thus, the statement is true for any multiple of eight.

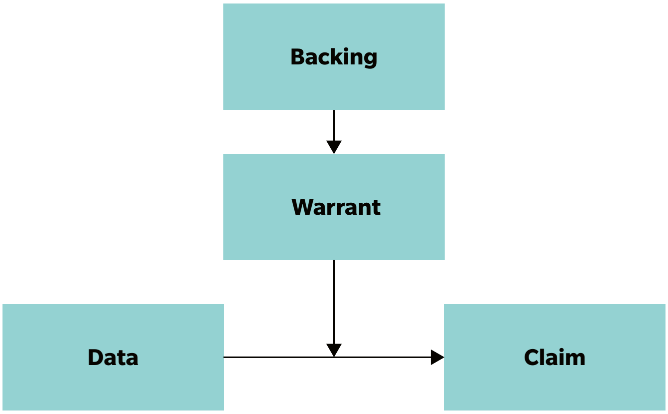
Structure of arguments

Toulmin's (1958) model of argumentation has been frequently used to analyse different aspects related to the structure of mathematical arguments (Krummheuer, 1995; Rø & Arnesen, 2020). We follow Krummheuer (1995) and consider an argument to consist of four elements: claim, data, warrant, and backing.

The *claim* is the conclusion of the argument. A claim is derived from *data*, that is, some known facts, and the justification for how the claim follows from the data is called a *warrant*. In some cases, a warrant needs further support, which is the *backing*. This layout is often presented in a diagram, as in Figure 16.1.

Figure 16.1

Toulmin's (1958) diagram with the four basic elements of an argument



In a mathematical argument, the warrant and backing should call upon axioms, rules of inference, and mathematical techniques. However, Toulmin's model is insufficient to reveal the mathematical validity of an argument. Instead, using the model provides insights into the argument's structure, allowing the researcher to study what characterises the warrants being made.

Furthermore, using Toulmin's model to analyse collective argumentation – that is, the construction of an argument during the class discussion – gives insight into the role of the teacher (Krummheuer, 2007; Yackel, 2002). Yackel (2002) points out that the teacher's role is to challenge students to identify what is known (data), what the rationales are (warrants and backings), and how statements follow from each other.

Communication moves supporting students' MR in discussions

We build on the inductive analyses of discussions made by Ødegaard (2023). In her work, teaching practices that support students' MR were identified by operationalising Jeannotte and Kieran's definition of MR; all teacher contributions were linked with the processes for MR they could potentially support. Thus, the more extensive practice of leading classroom discussions aiming to promote students' MR was decomposed into categories of moves reflecting MR processes.

The four categories were *moves for developing mathematical claims*, *moves for validating*, *moves for exemplifying*, and *non-MR moves*. In the first category, most moves were intrinsic to a specific MR process, such as 'Ask students to generalise.' Moves for validating held prompts and support to prove various types of conjectures, while moves for exemplifying were asking students to use a conjecture on an example or finding examples fitting a conjecture. Non-MR moves included more general moves such as revoicing and eliciting answers, asking for explanations, or confirmations of strategies.

Methods

We report from a study situated within a larger project on mathematical reasoning and proving in primary education (ProPrimEd[1]). We implemented role-plays of mathematical discussions aiming to promote students' MR in learning cycles. Twelve lessons were designed and implemented in the first mathematics course in the teacher education programme for grades 1–7 across three student groups of 30–40 PSTs each. The mathematical focus was mathematical reasoning across division, fractions, and measurement.

Each lesson started with the teacher educator introducing a classroom situation, focusing on a particular mathematical task and fictional students' work. The PSTs worked in groups, analysing the mathematical content of the task and the students' work. Next, they planned a discussion with the same fictional students, with a specified goal of promoting the students' MR. They used Ellis et al.'s (2019) framework of teacher moves as support during the planning. After planning, role-play was enacted in small groups or whole class.

The stated research question is answered through three intrinsic cases (Stake, 1995). For this study, these cases are groups of first-year PSTs in the 1–7 teacher education programme in their planning and execution of role-plays. Using Stake's (1995) terminology, the cases were given (rather than chosen) due to the groups' unique challenges during planning discussions promoting students' MR. Their challenges were identified as superficiality, overly demanding mathematical complexity, and unequal contributions by the PSTs in the group.

For further details, see separate sections introducing the cases in Findings. Data were collected through audio and video recordings of the groups' planning and enactment, which were subsequently transcribed and anonymised. While data from the planning phase form the basis for deciding which cases are to be further studied, the upcoming analysis and results are based solely on data from the groups' role-plays.

The analysis methods consisted of two analytic approaches, each detecting aspects of challenges that emerged in the PSTs' enactment of discussions

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[1] ProPrimEd – *Reasoning and Proving in Primary Education* is a collaboration between the Norwegian University of Science and Technology (NTNU) and Trondheim municipality, funded by the Norwegian Research Council. https://www.ntnu.edu/ilu/proprimed

aiming to support students' MR. The first analytic approach concerned structural aspects of the mathematical arguments built through the discussions. Using Toulmin's (1958) model, we thus studied how mathematical arguments were constructed, expressed, and elaborated on in the role-plays. More specifically, we identified mathematical claims stated in the discussions and searched for data, warrant, and backing leading to these claims.

The second analytic approach focused on the communication moves for eliciting and promoting MR performed by the PSTs acting as teachers. We built on the analyses by Ødegaard (2023) for teacher questions and students' responses. Indeed, two role-plays were analysed previously in Ødegaard (2023), while the third role-play was analysed deductively using the same set of codes.

Combining the two analytic approaches, we could determine whether the arguments produced were mathematically valid – that is, if they were grounded on valid and explicitly stated warrants, and if the discussions led to strengthened or more detailed mathematical arguments. Moreover, we elicited details on who produced the arguments (teacher, students, or both), and assessed whether the teacher's moves were instrumental in producing the arguments.

Findings

For each case, we provide an introduction to the task and the observed challenges for the given group during planning. This is followed by the analysis of the role-plays.

Introducing case 1: Superficiality during planning

We follow the PSTs Marius and Ane. During their planning, we observed challenges in terms of superficiality and a lack of in-depth discussion of the mathematical content. The given scenario was a 6th-grade teacher intending to make the students discover a relationship between the divisor and the quotient when the dividend is constant. Given a list of pairs of division problems below, the task was to determine which problem would yield the largest number, without performing the calculations. The fictive students were supposed to be supported in expressing and justifying a conjecture regarding the general relationship described above.

a) 24:12 or 24:4
b) 18:4 or 18:10
c) 5:1 or 5:0.5
d) 3:4 or 3:7
e) 238:15 or 238:16
f) 7:3 or 7:0.3
g) 12:0.2 or 12:0.01
h) 2.4:1.1 or 2.4:0.11
i) 21:0.3 or 21:0.0000001

The PSTs were asked to plan a discussion between a teacher and two students who had solved the problems but did not express the general relationship. Marius and Ane soon agreed that the conjecture is 'when the divisor is smallest, the answer is largest.' However, when discussing the task, they mainly engaged in off-topic talk. Moreover, they spent some time on which communication moves to use, but did not discuss the mathematical content in detail.

Marius said: 'I find this example [c)] hilarious. First you divide by a person, then you divide by half a person', but Ane and Marius did not seek to make further sense of division when the dividend is a rational number.

Analyzing case 1: Superficiality during planning leads to a superficial role-play

The analysis revealed that the discussion did not include an argument for the general conjecture. Also, the teacher did not probe into the students' reasoning. We present an excerpt of the role-play where Marius played the teacher, asking the student (Ane) for answers to a) to h).

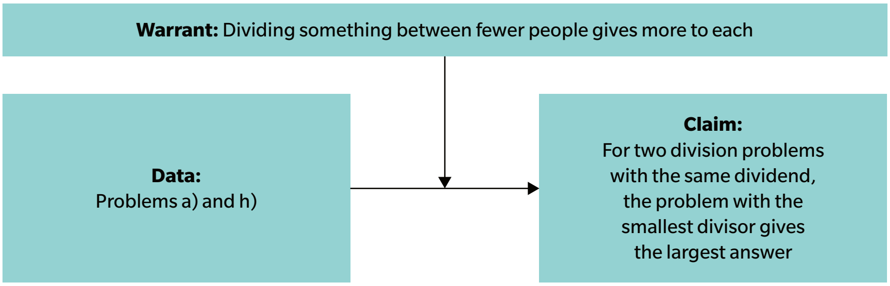
Marius: What have you found out for problem a)?
Ane: For a), I've found out that 24:4 gives the largest answer.
Marius: Yes, it's largest for 24:4, but what about problem h)?
Ane: For h), it's 2.4:0.11.

Marius further asked about a relationship between the problems, and Ane replied: 'When the divisor is smallest, the answer is largest.' Marius asked why, and Ane briefly explained using a practical context: 'When you divide something between fewer people, for example, they will get more each.' The argument is displayed in Figure 16.2.

The discussion continued with another pair of examples, but the structure of the argument remained the same as in Figure 16.2, with the warrant repeated, but not strengthened.

Figure 16.2

The structure of the mathematical argument developed in the role-play in case 1



The discussion was short, containing 11 utterances. Marius, acting as the teacher, made six utterances. He first asked for the student's answer to two tasks, and then for a relationship between the two tasks. He did not ask Ane to justify her conclusions. Next, Marius asked the student to validate the general conjecture, but with no attempt at highlighting relationships and properties. Finally, he asked the student to exemplify with another pair of division problems, which does not extend the justification already given.

We recall that the PSTs did not spend time on in-depth discussions of the mathematical content in their preliminary work. This was reflected in the role-play through the fast pace of the teacher's actions, and the restricted warrant for the claim. In particular, they did not adequately discuss the case of rational numbers. In the role-play, they used problem h), which includes rational numbers, seemingly without noticing that the justification provided by Ane does not make sense for this problem.

Introducing case 2: Too demanding mathematical complexity

In the second case, we follow Arya, Oline and Omar, whose main challenge during planning was the demanding mathematical content of generalising and proving that one divided by a fraction is the inverted fraction. The given classroom situation was a video on division of fractions (Boaler & Humphreys, 2005). The pupils use various model-based strategies solving $1:\frac{2}{3}$, before the teacher introduces a contextualisation, asking how many 2/3-foot-long pieces of lumber one foot of lumber can be divided into. We added a student, 'Martin' (not in the video), who said:

> *I thought that if I'd got 1:1/3, it would have been 3 because there's room for three-thirds in 1 foot of lumber. But we're to find 1:2/3, so we'll make pieces of lumber that are 2/3 of a foot long. Those pieces are twice as long as those that are 1/3 of a foot, so then it's half as many pieces as in 1:1/3. Then it's 3 divided by 2, which is 3/2. We get 3/2 pieces of lumber.*

The PSTs were asked to plan a whole-class discussion continuing where the video stopped, using Martin's strategy to develop and prove the general conjecture that 1 divided by a fraction is the inverted fraction (expressed as $1:\frac{a}{b}=\frac{b}{a}$). Oline and Arya worked together during planning, and expressed that they did not understand the mathematical content at stake:

> Oline :We are to make a general rule from what he [Martin] says. That's the stuff none of us understood (Arya laughs). So that's going to be exciting.

They planned to probe further into Martin's strategy, and to include other students, but they were unable to write the students' explicit answers to their questions. Thus, we consider the main challenge of Oline's and Arya's planning to be the struggle to grasp the mathematical content. The group entered the role-play with some questions and a general plan of the structure of the discussion, but with the students' input missing.

Analysing case 2: Mathematical challenges are reflected in the role-play

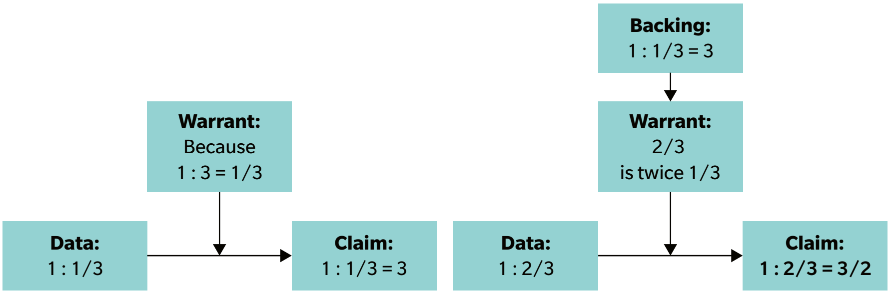
The analysis revealed that the arguments constructed during the discussion were incomplete. Arya took the teacher's role, Omar played Martin, and Oline played another student.

Although we do not have data showing Omar's participation in the preliminary parts of the lesson, we can assume that he, like the two girls, was unable to give a sufficient argument for the general claim. Arya, in the role of the teacher, consistently accepted the students' input and did not dwell on important mathematical aspects.

The discussion included several arguments. The first two of these involved Martin repeating his previous utterance, which was elicited by the teacher (Figure 16.3). The warrant is incomplete; it points to some property but does not explain the inference.

Figure 16.3

The structure of the first two mathematical arguments in case 2, in which the character Martin is repeating previous utterances



Arya did not dwell on Omar's explanations, but progressed by asking him what would happen if one divides 1 by 4/7. Omar answered:

> Omar If I use the same method, then I have to begin with 1 divided by 7, that is 1/7. Then, like last time, you get seven equal parts, so the answer is 7. And then we should have how many, 4/7. We know that 4/7 is four times as large.

Here, data and claim are centred around a single fraction computation. The warrant (why the answer is correct) is partly the vaguely described idea that 1:1/7 = 7 and 4/7 is four times 1/7. Arya tried to back Omar's warrant by adding: 'Yes, because it's three times bigger than, no four times bigger.' Omar answered: 'It is four times larger, so you can divide in 4, so the answer is seven divided by four, or 7/4.' This warrant points at some mathematical properties, but it does not suffice as mathematically valid.

Arya engaged another student, played by Oline, to look for a pattern in Martin's work (the computations were written on the board). After some hesitation, Oline said, 'I see that one divided by a fraction will be the inverse fraction', which is the conjecture the PSTs were asked to prove. However, the role-play ended here, as the PSTs went out of character trying to prove Oline's conjecture.

The role-play was about 30 utterances long, and about half were Arya's utterances. Several of them were short validations of the students' thinking. However, the general structure of the moves was that she started the discussion by eliciting the students' thinking before she focused on generalising the strategy: 'If you swap the numbers in a fraction with a different fraction. What happens then? If you, for example, take 1 divided by… 4/7.' Furthermore, she asked explicitly about the general pattern, which she eventually asked Oline to validate: 'It will always be like that. But why is that?'

Introducing case 3: Unequal contributions by the PSTs in the group

In Case 3, the challenge was unequal contributions by the PSTs Lilja and Karen during the group work, which led to them entering the role-play with different levels of mastering the mathematical content at stake. The fictional scenario was students arranging the following fractions from smallest to biggest:

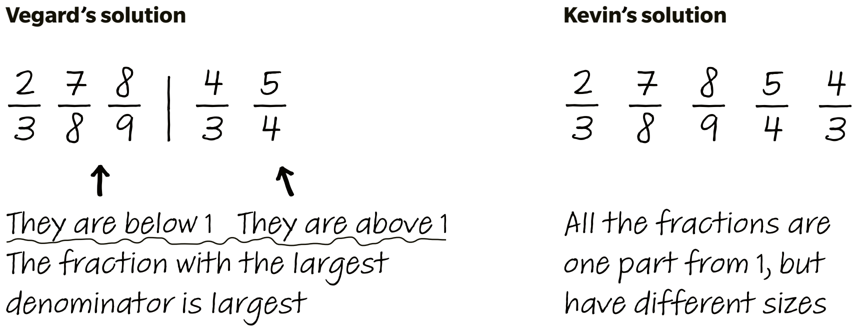
$$\frac{7}{8} \quad \frac{2}{3} \quad \frac{5}{4} \quad \frac{8}{9} \quad \frac{4}{3}$$

Moreover, the fictional students should identify what the fractions have in common, which can help compare fractions. The given fractions were all one 'part' away from 1 (e.g. 7/8 is 1/8 away from 1), and the size of this part can be used to determine the order of the fractions.

Two fictional students' works are displayed in Figure 16.4. The PSTs were asked to plan a discussion with each student, eliciting their thinking, and aiming to reach a claim that the 'missing parts' can be used to determine the order of the fractions.

Figure 16.4

Two fictional students' works studied in Case 3



Vegard's solution

$$\frac{2}{3} \quad \frac{7}{8} \quad \frac{8}{9} \; \Big| \; \frac{4}{3} \quad \frac{5}{4}$$

They are below 1 They are above 1

The fraction with the largest denominator is largest

Kevin's solution

$$\frac{2}{3} \quad \frac{7}{8} \quad \frac{8}{9} \quad \frac{5}{4} \quad \frac{4}{3}$$

All the fractions are one part from 1, but have different sizes

Lilja and Karen solved the task and discussed the students' work with two other PSTs. Lilja explained the solution of the task several times to the group, for example: 'This one [8/9] has nine parts, meaning it is only a small ninth left over, right? Here [7/8], it's 1/8, and 1/8 is larger than 1/9. That eight-part.' In contrast, Karen's contributions to the discussion were either off-topic or faulty reasoning, such as: 'But if you draw them [the fractions] and divide them into parts, this in nine [parts] and this in eight [parts], and colors eight and colors seven, they [the fractions] will be equal.' Moreover, Karen initially thought that Vegard's solution was correct, leading to guidance from Lilja. During planning, Karen's contributions often focused on general pedagogical aspects of teaching, while Lilja focused on the students' mathematical reasoning.

Analyzing case 3: Unequal contributions during planning lead to quality differences in teacher enactments

The analysis reveals two arguments with a similar structure, based on the idea of a missing part. However, the discussions differed in how the students were supported. When Karen played the teacher, she did most of the reasoning herself instead of promoting the student's reasoning. Lilja, on the other hand, elicited and promoted the students' reasoning.

In the first role-play, Karen took the role of the teacher, and Lilja acted as Vegard (see Figure 16.5). Their discussion consisted of 11 utterances. The teacher initiated the discussion by focusing on two fractions, namely, 2/3 and 7/8, and asked Vegard to draw a model. The discussion went on:

> Karen: Now you can demonstrate, with the model, whether it's true that 2/3 is less than 7/8.
> Lilja: (draws) So, now I've drawn 2/3 here and 7/8, and then the remainder is less coloured than that one.

Next, Karen asked Lilja to make an illustration for the fractions that were above 1. Lilja, with the support of Karen, concluded that Vegard's initial assumption on 5/4 being larger than 4/3 was wrong, 'because 1/3 that is extra here, is larger than 1/4 that is extra here'. Again, an argument was constructed for a comparison of two given fractions, where the warrant was based on comparing the part that makes the fraction 'one away from 1' and using a visual model. Karen made the final comment:

> Karen: So, we have found out you've been thinking correctly on that side with those [fractions] that are below 1, but you need to think a bit reversed when you come to that side which are above 1.

Applying Toulmin's model, the new claim is that, when you compare fractions one part away from 1, you need to use different strategies for fractions below and above 1. There is no explicit warrant supporting this generalised claim, maybe apart from an implicit, empirical warrant following from the two examples they had just discussed.

Karen made six utterances acting as the teacher. Two times, she asked Lilja to make a drawing to validate a specific comparison. Once, she asked 'What did you see now?', to support Lilja in rejecting her first ordering of 4/3 and 5/4. Moreover, Karen asked Lilja to explain what she observed using the drawing, as seen above. Finally, Karen made a summary of the student's thinking.

In the second role-play, also 11 utterances long, Lilja played the teacher, and Karen played Kevin (see Figure 16.5). Lilja started by asking about the conjecture, which Karen read from Kevin's worksheet. Lilja repeated the conjecture and asked how the student used it to order the fractions. Karen repeated the ordering: 'I chose to put 2/3 at the bottom, then 7/8, 8/9, and then I chose 5/4 and 4/3.' Lilja asked Karen to elaborate on her reasoning, and Karen explained:

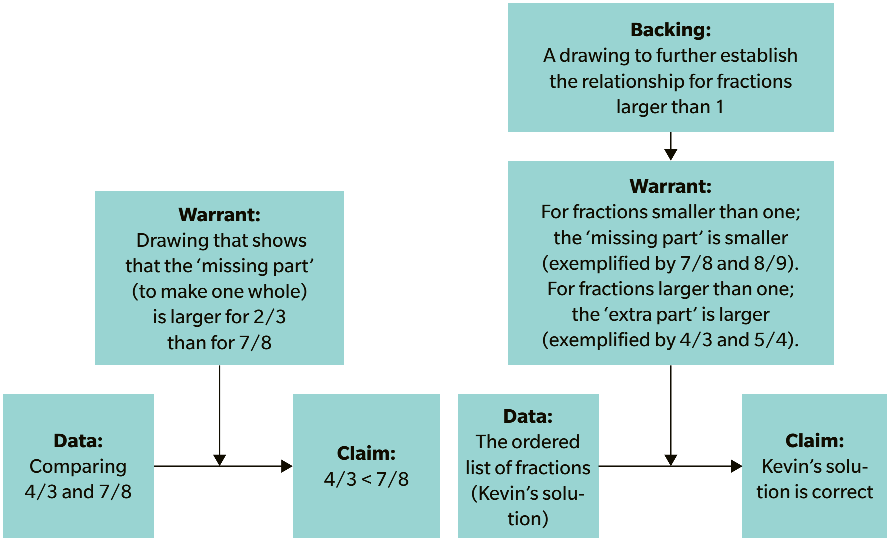
> Karen: …if you divide that one into eight and nine parts, then it's like one part is smaller, so, therefore, they get bigger in a way, because you miss a smaller part to make one whole.

Lilja asked about what happens with fractions larger than 1, and Karen explained how to compare 4/3 and 5/4. Lone asked her to make a drawing following her explanation, which Karen did, and the discussion ended.

The two structures of the two arguments developed in the role-plays in case 3 are as below.

Figure 16.5

The structure of the mathematical argument developed in the first (left) and the second role-play (right) in case 3



Lilja used her first three utterances to elicit Kevin's thinking. She asked for the conjecture, which she repeated in clarifying language, before asking how the student solved the task. When Karen just gave the ordering of the fractions, Lilja requested her to explain her reasoning. After Karen explained her reasoning for fractions below 1, Lilja asked about fractions above 1. Finally, Lilja asked for a drawing to illustrate the comparison of 5/4 and 4/3, and Karen's explanation ended the discussion.

The two role-plays included many of the same elements: comparisons of fractions above and below 1 based on the comparable sizes of the 'missing/extra' part. Yet, there were quality differences in teachers' enactments, which we argue originate from the PSTs' unequal contributions to the group.

In the first role-play, the teacher provided the main argument (based on Vegard's examples) with only an implicit, empirical warrant. Lilja stayed close to the character Vegard and needed the teachers' support before correcting the mistake in Vegard's solution. However, Karen did not seek to extend Vegard's reasoning and provided the explanations herself.

In the second role-play, the student provided the warrants in response to the teacher's (Lilja's) prompts.

Discussion

This study contributes to research on practice-based approaches to teacher education by investigating the enactment phases of the learning cycle in light of challenges identified during planning. In addition, the presented findings display characteristics of the enactments regarding positioning students as sense-makers and reaching a valid mathematical argument. The analysis of the structure of the mathematical argument constructed through the discussions offered more profound insights into the nature of the discussions played out.

In Case 1, the PSTs did not plan the discussion sufficiently. As a result, the enactments lacked attention to properties and relationships, and the teacher did not follow up on students' utterances. Here, the resulting mathematical argument was incomplete, and the students' contributions were not the source of further discussion.

In Case 2, the PSTs struggled with solving the task themselves, and, similar to Mendes' (2022) study, they also faced challenges in the enactments. Here, the enacted discussion was characterised by incomplete arguments, and teacher moves that did not build upon the students' contributions. Thus, Case 2 demonstrates the importance of predicting students' responses during planning, as Buchbinder and McCrone (2020) highlighted.

Case 3 provided insight into the importance of mathematical knowledge through two different PSTs' contributions during planning and enactment. The PST, who expressed challenges in understanding the mathematical content, enacted a teacher-centred discussion and did not ask the student for explanations. In contrast, the PST, who appeared more knowledgeable in mathematics, enacted a student-centred discussion.

Participating in the learning cycle allowed the PSTs to practise important characteristics of leading mathematical discussions. Studying the teachers' communication moves revealed that the overall structure of the discussions was usually adequate, as they followed Ellis et al.'s (2019) recommendation of first eliciting, then responding, and further extending students' reasoning.

For example, in Case 2, the teacher elicited Martin's thinking, and she sought to make him generalise it before turning to the other student(s) to establish a general conjecture from the pattern observed in Martin's examples. Finally, she tried to make the students justify the conjecture.

In particular, the discussion held a more appropriate structure than the initiate–response–evaluate pattern (for more details, see Drageset, 2014), or the pattern observed by Drageset (2014), where teachers followed up students' utterances with closed questions, and simplified the tasks such that the teacher's wording shaped the students' responses.

Case 3 shows an example of a role-play where the teacher successfully centres the student's thinking and positions her as a sense-maker.

Despite the observed opportunities for PSTs' learning to teach, previous research shows that leading discussions is a complex practice (Shaughnessy et al., 2019; Buchbinder & McCrone, 2020; Stylianides et al., 2017), that predicting students' responses is challenging (Buchbinder & McCrone, 2020), and, not the least, that reasoning and proving in mathematics is difficult (Stylianides et al., 2017). When using role-plays, it is essential to be aware of these different dimensions of complexity.

In Case 2, the complexity of the mathematical content led to challenges in both planning and enactment. Predicting that generalising the division of fractions would be demanding for the PSTs, we provided a strategy from which an argument could be constructed. However, the fictive student Martin's single utterance was insufficient for the PSTs to utilise the strategy. More robust scaffolding, such as providing additional dialogue with in-depth strategy discussion, could have helped the PSTs build their argument.

Moreover, role-play is an approximation to practice in which the PSTs are responsible for planning and enacting, and where the teacher educator does not guide the PSTs closely. Thus, challenges of superficiality during planning and enactment, as in Case 1, can be expected to occur.

The PSTs could have discussed the mathematical content in more detail if a teacher educator had supported them. However, having a teacher educator take a more prominent role requires more resources for implementing the learning cycle. Consequently, it might be implemented less frequently. As we believe that PSTs need repeated experiences with enacting practices, we call for continued effort to find ways to support PSTs adequately during planning and enactments.

To conclude, we emphasise that role-plays provided the PSTs opportunities to sequence and enact moves that promoted MR, thus making role-plays of mathematical discussions a promising approach in teacher education. Yet, the challenges we identified in PSTs' planning impacted the enactments of the role-plays. Thus, more innovations in teacher education are needed to address challenges related to MR, and other mathematically complex content.

In particular, we encourage research initiatives that focus on integrating the mathematical content (e.g. MR), and the didactical content (on orchestrating discussions) the PSTs engage with, within a practice-based approach to teacher education.

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