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Self-efficacy in Mathematics and Teaching Mathematics in Novice Elementary Pre-service Teachers

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Abstract

This paper investigates how measures of self-efficacy and beliefs can be used as tools for understanding pre-service teachers' (PSTs') entry into the teaching community of practice, where their identity as a teacher of mathematics can be partially explored in terms of self-efficacy in teaching mathematics (SETM) and mathematics self-efficacy (MSE) as joint indicators of their developing competencies. Analysis of the responses of 191 novice elementary PSTs in a Norwegian university college to questionnaires comprising 50 4-point Likert scale items shows that MSE and SETM are correlated. Combined with analysis of interviews with five PSTs investigating their modes of identification with the practice of teaching mathematics, this paper aims to give a more nuanced picture of the different ways in which PSTs identify as future teachers by addressing the following questions: What are the connections between novice PSTs' perceptions of their own subject knowledge and their self-efficacy as a potential teacher of mathematics? What are the implications for the identity work these PSTs need to do? Given the importance of PSTs' developing self-efficacy in teaching during initial training, these descriptions have the potential to inform teacher educators in tailoring training to meet different PSTs' needs, including their starting point for building self-efficacy in teaching mathematics.

Keywords: *mathematics self-efficacy, self-efficacy in teaching mathematics, pre-service teachers, elementary teacher education.*

Background

In 2010 the Norwegian Ministry of Education and Research adopted new regulations for initial teacher education, replacing a generalist training across the school years with distinct pathways for grades 1-7 (age 6-12) and 5-10 (age 10-15). While students training for grades 5-10 can opt to specialise in mathematics, Norwegian, English or science, pre-service teachers (PSTs) on the elementary teacher training programme ("Grunnskolelærerutdanningen", abbreviated to GLU 1-7) must pass a compulsory 30 credits mathematics course. As a group who must teach mathematics, their previous experience with mathematics is clearly important; however, Smestad, Eriksen, Martinussen, and Tellefsen (2012) found that while 74% of 117 PSTs at GLU 1-7 reported having positive experiences with mathematics at elementary school, only 40% reported positive experiences at upper secondary schooling. These recent experiences are likely to be fresh in the novices' minds, as most PSTs enter their teacher training immediately after completing upper secondary school. Given the emphasis on mathematics in their training and their future career, this finding raises issues regarding the potential impact of negative experiences on PSTs' beliefs about teaching mathematics.

Students enter a university programme with a set of prior beliefs about mathematics and how to teach and learn it; these beliefs are considered difficult to change (McLeod, 1992; Philipp, 2007). Drawing on the concept of personal epistemologies (Pintrich, Hofer, & Pintrich, 2002), De Corte, Op't Eynde, and Verschaffel (2002) describe a common personal epistemology when it comes to mathematics: "Mathematics is associated with certainty, and with being able to give quickly the correct answer; doing mathematics corresponds to following rules prescribed by the teacher; knowing math means being able to recall and use the correct rule when asked by the teacher; and an answer to a mathematical question or problem becomes true when it is approved by the authority of the teacher" (p.305). Teachers holding such traditional views of mathematics are more likely to engage in correspondingly non-interactive teaching practices (see Stipek, Givvin, Salmon, and MacGyvers (2001)). Traditional views of mathematics are common, and students entering a teacher training programme are highly likely to have experienced them at school; this is important since, as Prescott and Cavanagh (2006) note, even if PSTs' memories from their own schooling are not completely correct, what they do remember may be important indicators of what they think mathematics teaching is and should be. Furthermore, Arvold (2005) argues that PSTs experience and interpret their teacher education through the lens of their prior experience of being taught mathematics.

Beliefs about mathematics are also likely to be connected to one's sense of one's ability to do mathematics. This is captured in Bandura's (1986) concept of self-efficacy, which he considers to be more predictive of future performance than the more global indicator 'confidence'. Self-efficacy is to be looked upon as a two-dimensional construct, i.e. a belief about action and outcome, 'outcome expectancy'; and a personal belief about one's own ability to cope with a task, personal 'self-efficacy' (Bandura, 1986). Applied to mathematics, Hackett and Betz (1989, p. 262) define mathematics self-efficacy (MSE) as "a situational or problem-specific assessment of an individual's confidence in his or her ability to successfully perform or accomplish a particular [mathematical] task or problem".

Since the 1960's researchers have explored the relationships among teacher characteristics and student achievement using various measures (Hill, Rowan, & Ball, 2005). However, Woolfolk and Hoy (1990) noted that while few consistent relationships between the characteristics of teachers and the learning outcome of pupils have been identified, self-efficacy in teaching is an exception to these general findings (p. 81). Tschannen-Moran and Hoy (2001, p. 783) defined self-efficacy in teaching as a teacher's "judgment of his or her capabilities to bring about desired outcomes of student engagement and learning, even among those students who may be difficult or unmotivated". Like general self-efficacy, self-efficacy in teaching can be regarded as a two-dimensional construct that includes personal teaching efficacy, and teaching outcome expectancy (Enochs, Smith, & Huinker, 2000). It is conceived to be subject-matter specific (Tschannen-Moran & Hoy, 2001), and thus a subset of self-efficacy in teaching, self-efficacy in teaching mathematics (SETM), is a measure of the efficacy to teach mathematics (Esterly, 2003, p. 13).

SETM is influenced by teachers' own MSE, their mathematical beliefs (Briley, 2012; Esterly, 2003; Morselli, 2005), and their past experiences as learners of mathematics (Brown, 2012). Research indicates that PSTs' mathematical beliefs do not change during teacher training (Esterly, 2003), while self-efficacy in teaching develops mainly during teacher training (Hoy & Spero, 2005; Mulholland & Wallace, 2001; Smith III, 1996), tending to decline during the first year of teaching (Hoy & Spero, 2005). These findings underline the importance of building SETM during teacher education and the need for teacher educators to understand more about the experiences, beliefs and levels of self-efficacy which their novice PSTs bring from earlier schooling. This paper addresses these issues by exploring the identities of novice elementary PSTs in order to enable teacher educators to understand more about their needs in developing SETM during teacher training. Focusing on the relationship between SETM, MSE and prior experience, it investigates the range of starting points from which PSTs' developing identities as mathematics teachers emerge, with implications for the identity work they need to do, and be supported in, during their teacher education.

Theoretical Framework

To better understand the complexity of being a novice elementary PST, and the circumstances in which a PST's initial SETM will develop, it is helpful to frame this paper within Wenger's social learning theory (Lave & Wenger, 1991; Wenger, 1998), and its conception of learning as a social activity derived from active engagement in a practice. In particular, it enables us to theorise individual trajectories and development of identities as these relate to competences in mathematics teaching. The addition of insights from Biesta (2012) extends our understanding of the relationship between such competences and PSTs' developing practice as teachers in terms of their growing awareness of how to *make judgements*: as he says, a teacher who possesses all the competences teachers need but who is unable to judge which competence needs to be deployed and when, is a useless teacher (Biesta, 2012, p. 42).

The landscape a novice elementary PST has to navigate presents a complex picture of mainly two concerns. First, for most PSTs, their most recent experience with schooling will be as a pupil in upper secondary school. This past experience is part of their identity formed within processes of participation (for example as active learners asking questions, or as passive learners answering questions) and reification (of artefacts such as tests, textbooks, and so on), which are intertwined over time in a particular practice (Wenger, 1998, p. 87). These situated experiences are coloured by different teaching traditions in practices with different enterprises (different objects of learning: learning rules to get right answers to pass a test, versus engaging with understanding mathematical concepts) and repertoires (for instance how to do mathematics, which words to use and which routines to follow). Their perception and experiences of the ways in which the experts (mathematics teachers) in classroom communities of practice have contributed to their development of mathematical competences will influence PSTs' developing identities as learners of mathematics. This past experience as a participant colours who they are and is not something that can be 'turned on and off' (Wenger, 1998, p. 57).

One way to capture the nature of these mathematical competences and their role in developing teacher identities is by paying attention to PSTs' MSE and SETM. This leads to the second concern: By crossing the boundaries between the school and university college communities of practice, novice PSTs have to move from no longer seeing themselves (and being seen by others) as pupils, to being seen as PSTs. This involves navigating between their roles as students (from the teacher educators' and mentors' view), as student-teachers (by pupils during school placement) and as prospective teachers (on the basis of their choice of attending a teacher training programme). There is a set of criteria and expectations to handle in order to achieve membership: understanding what matters, being able to engage productively and using appropriately the repertoire of resources available in a practice (Wenger, 2012, p. 2). They are no longer in a familiar practice, and they have to shape their trajectory on their way to developing identities as prospective teachers. By exploring PSTs' initial MSE and SETM, we can better understand their starting point for navigating this landscape of practices.

Their participation in this new constellation of practices can be described in terms of what Wenger (1998) characterised as three distinct modes of belonging, and later as modes of identification (Wenger, 2012): engagement, imagination and alignment. *Engagement* is the active involvement in practice, while *imagination*, on the other hand, involves standing back from the world and seeing oneself in it as a part of the whole picture (Wenger, 1998, p. 176). *Alignment* is all about doing what it takes to play a part in the practice. Identifying PSTs' different modes of identification is useful for making sense of their developing identities as prospective teachers of mathematics, and provides a framework for addressing the central research questions of this paper.

Research questions

The complex demands on PSTs in terms of their multiple roles as students, student-teachers and prospective teachers can be captured in terms of how PSTs engage with mathematics as a school subject: it is no longer sufficient to be able to *do* the mathematics themselves. As PSTs and prospective teachers a new dimension is added in terms of a set of new competences;

one should be able to *teach* mathematics and enable other people to *do* and preferably *understand* mathematics. This newly added dimension of how to teach mathematics is captured in the concept of SETM which is influenced by beliefs, previous experience as learners, and MSE. SETM is indicative of the identity work that individual novice PSTs need to do on their trajectory towards becoming a teacher of mathematics. Modes of identification are thus applied here alongside SETM to answer the research questions addressed in this paper:

What are the connections between novice PSTs' perceptions of their own subject knowledge and self-efficacy as a potential teacher in mathematics? What are the implications for the identity work these PSTs need to do?

Method

Participants and context

This paper reports on results from the initial data collection of a larger project tracking novice PSTs through their first 2 years of training. The entire cohort of 2013, 191 novice PSTs at Oslo and Akershus University College (average age of 22.5 years, and about 20% men), completed a threefold questionnaire capturing MSE, SETM, and mathematical beliefs early in their teacher education programme, thus ensuring that their recorded beliefs were based solely on their previous experience as pupils in school. In addition, the quantitative data were supplemented by qualitative data from interviews. PSTs who were interested in being part of the overall project were invited to indicate this on their questionnaires and were later invited for on-going in-depth study. Ten PSTs were subsequently interviewed, three men and seven women. This paper reports on analysis of the first interview with five of the ten PSTs, selected for inclusion here on the basis of their questionnaire responses, as explained below.

Instrument

The instrument is threefold. The MSE element is an adaptation of an instrument originally developed by Pampaka, Kleanthous, Hutcheson, and Wake (2011) and validated using Rasch analysis (Bond & Fox, 2007). It requires respondents to say how confident they would be using mathematics to solve 30 different problems, using a 4-point Likert scale with answer categories "Not confident at all", "Not very confident", "Fairly confident" and "Very confident". They are not asked to actually solve the problems. The tasks in the original English instrument were designed to measure MSE as a learning outcome of post-compulsory mathematics education in the pre-university phase (Pampaka et al., 2011). For the current study, they were translated into Norwegian, and mapped onto the Norwegian upper secondary school curriculum in order to ensure that novice PSTs should be able to do them.

The SETM element was developed and validated (also using Rasch analysis) by the author of this paper (Bjerke & Eriksen, in progress) and requires respondents to say how confident they are helping a child with 20 different tasks, using a 4 point Likert-scale with answer categories "Not confident", "Somewhat confident", "Confident" and "Very confident". Novice elementary PSTs have no teaching experience, and all they can say about teaching is based on their own experience as pupils. Therefore, the SETM element of the instrument consists of tasks with the setting "helping a child" and aims to measure their initial SETM without demanding any experience. This might sound like a paradox, but we do all have some initial thoughts about teaching on the basis of our own experience at school. Thus the instrument is designed to be sufficiently concrete and intuitive to tap these initial thoughts, and is worded in a way familiar to PSTs. In addition, the SETM element of the instrument addresses familiar mathematics that the PSTs are going to teach when they enter schools as teachers, even if some of the tasks might be unfamiliar to those PSTs who have been taught by teachers holding traditional views of mathematics as described by De Corte et al. (2002).

Ten of the 20 tasks are based on instrumental understanding – 'rules without reasoning' as described by Skemp (1976), later referred to as 'Rules'. "Calculate 750:25" is an example of such a task, which simply asks for calculation without any further explanation. The other 10 tasks are based on what Skemp calls relational understanding, requiring "knowing both what to do and why" (Skemp, 1976, p. 20), later referred to as 'Reasoning'. "Explain that division doesn't always make things smaller" exemplifies this kind of task. It is pointed out to the PSTs that when the verb "explain" is used in the tasks they are asked to help the child to be able to explain. The labels 'Rules' and 'Reasoning' are used here in order to connect the idea behind the tasks to the belief-statement in the third element of the instrument.

The third element of the instrument consists of 21 statements relating to mathematical beliefs, 10 tapping instrumental understanding and more transmission teaching beliefs ('Rules'), and 11 tapping relational understanding and more connectionist approaches ('Reasoning'). Responses options use a 4 point Likert-scale with categories "Disagree entirely", "Disagree somewhat", "Agree somewhat" and "Agree entirely". The belief element of the instrument and its two underlying constructs 'Rules' and 'Reasoning' were developed and validated by Drageset (2012) using 365 in-service Norwegian elementary teachers. For the purposes of this paper this third element of the instrument is only referred to within the interview analysis, and is not analysed separately.

Interviews

The semi-structured interviews took place six weeks after PSTs completed the questionnaire, and were intended to capture both their initial thoughts on being PSTs and their reflections on why they had answered the questionnaire in the way they had. This enabled a more detailed investigation of the reasoning behind their initial MSE and SETM responses, and provided an opportunity to explore more about their mathematical beliefs and their relation to their prior experience as mathematics learners. Combined with the questionnaire scores, these data provided an opportunity for triangulation, and enabled further insights into the complex relationship between their mathematical beliefs, MSE and SETM, and the range of PSTs' starting points. Based on their score combinations on the MSE and SETM element of the instrument, five of the ten interviewed PSTs were picked for the analysis in this paper in order to capture the range of positions generated by different combinations of MSE and SETM. While the use of a small self-selecting sample as the basis of a typology is potentially problematic, further selection of these five on the basis of an analysis of their location within the overall pattern of the whole cohort's questionnaire responses provides a systematic rationale. The selection of the five is explained in the results section below.

Analysis

The data analysis took place in three steps. First, the 191 responses on each of the SETM and MSE elements of the instrument were analysed using the Rasch Rating Scale Model (RSM). RSM supports the construction of a genuine interval estimate for each of the constructs, so that both items and persons are measured on the same scale. Consequently it enables reporting both of person estimates (here, the higher the estimates, the more evidence of the presence of MSE/SETM) and also item estimates (here, the higher the estimate, the more MSE/SETM is needed to endorse it). The analysis provides each of the PSTs with one SETM person estimate and one MSE person estimate, and at the same time it associates each of the items in the two elements of the instrument with an item estimate. Second, the estimated person values were used in order to check if the items in each of the elements of the instrument worked together to measure a single underlying one-dimensional construct. Both MSE and SETM appeared as solid constructs, measured as 0.93 and 0.89 respectively using Cronbach's Alpha. Third, interviews with the five chosen PSTs were analysed in terms of Wenger's modes of identification and issues from the previous literature, such as participation, reification and the formation of a repertoire. This analysis also took into account the third element of the instrument consisting of statements relating to mathematical beliefs.

Results

The initial results draw on the SETM and MSE measures calculated by Rasch analysis for each of the PSTs, and form the basis of the remaining analysis. Next, the analysis focuses on the relationship between novice PSTs' perception of their own subject knowledge (as in MSE) and their self-efficacy as a potential teacher in mathematics (as in SETM). This analysis results in identification of some outliers from the main trend who are commented upon, before moving on to the main findings, organised in terms of two categories of PSTs: those who fit the correlation between MSE and SETM, and those who do not. This main analysis section focuses on five PSTs who are selected to present these two categories, and combines questionnaire data from all three elements of the instrument, and interview data.

Rasch data

Since the RSM model measures both items and persons on the same scale it is possible to establish a person's probable answer on any item. This means that if we have a measure for each item in an instrument *and* for a person who has responded to the instrument, the measures can be compared. If for instance a PST's estimated MSE measure is less than half of the items in the instrument, this means that this particular PST is unlikely to report feeling able to do items with a higher difficulty estimate than his person estimate.

Figure 1 gives the spread of perceived difficulty for each item in each of the two measures and will later be compared with the PSTs' person estimates. The scales are analysed separately, and the MSE and SETM measures are not to be directly compared.

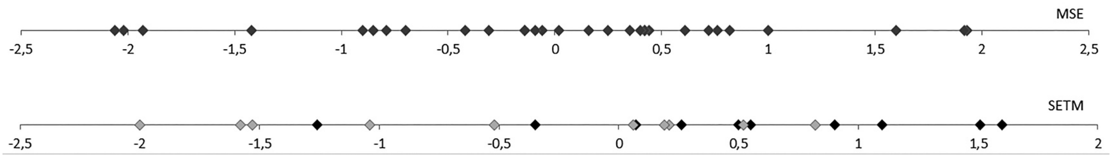

Fig 1. Item estimates for the MSE instrument and for the SETM instrument. Those items further to the right are those which the 191 PSTs find hardest to endorse. On the SETM scale the grey items are 'Rules' and the black items are 'Reasoning'. Thus 'Reasoning' is seen as more difficult on this measure.

Relationships between MSE and SETM

In order to investigate the relationship between MSE and SETM, correlation was calculated by Spearman's rho, since a Shapiro-Wilk's test ($p>.05$) revealed that the constructs are not normally distributed. As previous research predicts, the test showed that there is an overall moderate correlation between MSE and SETM (0.58) with a medium effect size ($\eta_p^2 = 0.49$), calculated using Cohen's criteria.

The scatterplot in Figure 2 confirms the tendency given by the correlation; the higher MSE, the higher SETM. But even though the measures are correlated and one could expect PSTs with high MSE to have high SETM as well, the picture is more complex. There are some reversals of this trend in the rankings within each of the two scales which are of interest for this study. For example, some PSTs might have a low MSE measure compared to the rest of the cohort, which would lead us to expect a similarly low SETM, but in fact they may have a high SETM measure. Of the ten PSTs who were interviewed, five were identified as representative of particular points in the overall correlation picture. Looking more closely at each enables us to understand more about the PSTs' profile, contributing to our understanding of the range of PSTs that teacher educators will encounter.

The following analysis focuses on five PSTs numbered in the scatterplot in Figure 2. Based on the spread shown, the correlation line and the calculated means for MSE (-1.05) and SETM (0.55), these PSTs were initially picked in order to represent two different main trends: those fitting the correlation (nearly on the correlation line) and those not (further away from the correlation line). The analysis explores their answers on the instrument alongside the interview data.

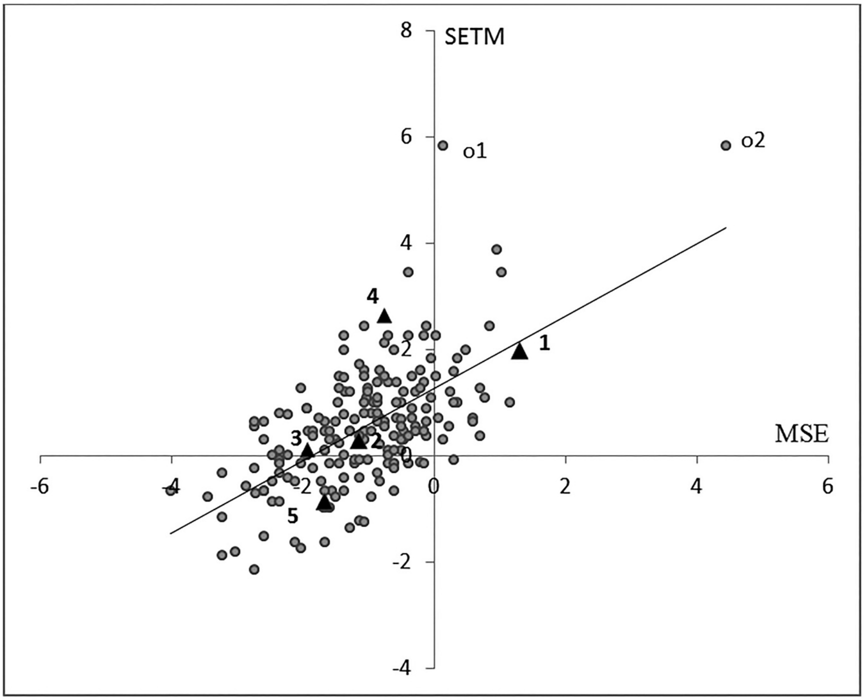

Fig 2. Scatterplot showing relations between each PST's MSE and SETM. PST1 – 5 are marked by triangles and their respective number. o1 and o2 are outliers.

Outliers

As Figure 2 illustrates, the majority of PSTs cluster together within a rectangle spanning the line of correlation. However, two outliers (o1 and o2) fall far outside of this cluster, indicating that they are not measured well in terms of MSE and SETM. o2(coordinates (4.43, 5.83)) is estimated to perform better than any of the tasks can measure, as can be seen from observing where 4.43 and 5.83 fall in the first and second lines respectively in Figure 1. This particular PST is therefore not well measured by these two elements of the instrument and should be excluded from the analysis. The same goes for outlier o1, who, while measured well for MSE (0.12) is not measured well by the SETM element of the instrument (5.83: he finds the tasks too easy).

PSTs fitting the correlation

Three PSTs were selected as representatives of the correlation: one best described as a PST with both high ('H-high') MSE and SETM measures (PST 1, HH), one with medium ('M-medium') MSE and SETM measures

(PST 2, MM) and one with low ('L-low') MSE and SETM measures (PST 3, LL). This selection ensures a good spread among those representing the correlation; as we shall see, the analysis reveals considerable differences in the trajectories they describe in terms of their modes of identification in previous practices and their navigation of the new landscape of practices.

PST1 (HH) has what she describes as a good relationship with mathematics and is very positive about being a teacher of mathematics. She ticks "Confident" or "Very confident" on 23 of the 30 tasks in the MSE element of the instrument, stating that if she had her rulebook (a book for writing down mathematical rules which is commonly used in upper secondary school in Norway and is allowed in tests and exams), then she would have been able to solve most of them. The rulebook can be seen as a reification of her knowledge, on which she heavily relies. She describes herself as a confident person. This picture is consistent with her MSE measure of 1.3 – fairly high compared to the average (-1.05). There are two issues to take into account when investigating her identity as a learner: First, she explains that she has never been afraid to ask the teacher *why* rules work. She is curious and disagrees strongly with the belief statement: "What you are able to do you also understand". Engaging in this way has enabled PST1 to develop her mathematical competences and this appears central to her identity as a learner of mathematics. Secondly, however, she also says that it is important to cram, since "it has done the job for me! … Mathematics is essentially a stack of rules … In order to get an answer I'm happy to follow these rules". Based on her account of her past experience and her responses to the belief statements, it seems that she has steadily followed a trajectory through mathematics of relying on rules, suggesting an identity of *alignment* despite her apparent engagement with understanding what lies behind them. It is hard to come to a conclusion regarding the implications her mode of learning has for her mode of identification as a PST and a prospective teacher. She has a high self-efficacy both in mathematics and in teaching mathematics, and we might conclude that she is therefore an 'ideal' novice PST. However, she does not reveal much about what kind of teacher she aims to be. Her emphasis on following rules suggests that she is still anchored in an identity of a learner of mathematics, and that she has considerable identity work to do towards becoming the teacher she wants to be.

PST2 (MM) says he enjoys mathematics and describes himself as an interested pupil who experienced a problem-free journey through elementary and lower secondary school. He achieved good grades "without trying hard", resulting in being what he describes as a "listener". He got used to being a “listener” and thought he could continue being one in upper secondary school as well. But upper secondary school demanded something different from him, and he remembers that he had to work harder and participate more actively as the mathematics got tougher. He says that he did not like that, which perhaps explains why he ticked “Not confident” or “Somewhat confident” on 23 of the 30 tasks in the MSE element of the instrument.

His responses on the SETM element of the instrument do not reveal anything in terms of rules or reasoning. He explains that his answers depend on the themes in the tasks, not whether the tasks ask him to explain or not, partly indicating that he does not notice the different demands of the tasks. However, he states that in his first weeks as a PST he can sense different approaches to teaching mathematics than those he has previously been exposed to (blackboard teaching and solving tasks in his notebook), saying he is open to these different approaches. He says he has already been influenced by teacher education; he sees that using concretes and creating more realistic problems is ’the way to go’.

Applying Wenger’s concept of modes of identification, we can interpret PST2’s story as an indication of a move from an aligned mode as learner with an identity as a “listener” to a mode of identification as PST best described as imagination: he stands back and reflects on the basis of new knowledge on his own previous situation as a learner. It seems that encountering difficulty in his own schooling has enabled him to reflect in useful ways which affect the way he wants to develop in his trajectory towards being a teacher of mathematics.

PST3 (LL) talks about competences gained in mathematics during elementary school, in terms of having a good *understanding* of mathematics which continued during lower secondary. He has a SETM measure of 0.11, which is fairly low compared to the rest of the cohort. He is “Not confident” that he would be able to help a child calculate multiplication involving decimals and addition involving fractions, explaining this in terms of the amount of time that has passed since he was involved in this kind of calculation. In upper secondary school he was not concerned to do well anymore and he has a comparatively low MSE measure of -1.93 which fits well with his comment that he gave up on mathematics in upper secondary school. Taking a closer look at his actual answers, he has ticked “Not confident” on 19 out of 30 tasks. He says that he did not understand the use of the rulebook that everyone relied on in upper secondary, since in his opinion the rulebook did not make them *understand* mathematics; “There is a very big difference between mechanically solving problems in mathematics and *understanding* it."

When applying Wenger's concept of mode of identification, PST3's process of identification can be divided into three phases. The first two concern his identity as a learner. First his mode of identification can be looked upon as *engagement* during elementary and lower secondary school, since he appeared to participate in a practice where he was offered opportunities to *understand* mathematics. But in upper secondary school he turned away from his long-standing membership of this practice and appears to have formed an identity of *non-participant* in the subsequent practice which focused on rules; he did not understand the use of the rulebook that everyone relied on and denounced the reification of knowledge that was pointed out as essential. He did not identify with this practice, and is therefore best described as a non-participant in the sense of marginality (taking a distance, moving away from full participation) or in terms of what Wenger (1998, p.155) labels an outbound trajectory.

But despite this past experience with mathematics during upper secondary school, he has no objection to mathematics. Standing back and reflecting upon what he has been offered during upper secondary school, he says that he would like to be a teacher who makes pupils "understand the logic behind". In this third phase of imagination (as a way of identification), his reflections enable him to initiate the identity work needed to become the sort of teacher he wants to be.

PSTs not fitting the correlation

The correlation between MSE and SETM is only medium, indicating that not all PSTs will align with it: PSTs placed either far to the left or far to the right on the first line in Figure 1 (MSE measures) might not be placed correspondingly on the second line (SETM measures).

PST4 (MH) describes herself as a person who always has been fond of mathematics. She has an MSE measure of -0.76 (close to the mean) and has ticked "Not confident" on 15 of the 30 tasks: "I guess I didn't really understand it at the time I engaged with this kind of mathematics". According to her belief-statements responses and what she says during the interview, understanding mathematics is important to her. Based on the experience of her own schooling, she describes a good mathematics teacher as one who can explain and help both those who are strong and weak in mathematics. Her SETM score of 2.64 is fairly high compared to other PSTs: she ticks "Very confident" on all but six Reasoning-tasks. She explains that her different answers on the Reasoning-tasks depend on whether or not she has tried to explain these kinds of tasks before. She says: "it is more complicated with these tasks because in addition to explaining one must be able to familiarise oneself with the limits of what the pupil is able to do, and how the pupil thinks". Statements like this reveal a reflective PST with a mode of identification as a PST best described as imagination. She sees the "regime of competences" (Wenger, 2012, p. 2) in terms of a perception of what matters (understanding mathematics) and an idea of how to use an appropriate repertoire needed in order to become what she labels "a good teacher". She reflects back on herself as a participant in a practice not as focused on understanding as she would like, imagining what could be done differently.

In contrast to PST4, PST5 (ML) has one of the lowest SETM scores: "I am rusty on all of this!" She stresses that she needs to know this before she can teach it to someone else, or "it will get messy". Her MSE is just below the mean (comparing closely to just above the mean for PST4), and during the interview she explains that she felt stupid and frustrated when completing the form. Even though PST4 and PST5's MSE-scores are fairly close, PST5's identity as a learner of mathematics is completely different. By her own account, she likes mathematics when she is able to do it, but when she does not 'get it', she does not like it. This and related statements make her story more emotional as she tends to describe her relationship to mathematics as dependent on how doing it makes her feel. It is hard to make her reflect on her own experience as a learner of mathematics: she thinks she learns best by doing practical exercises and using manipulatives, not just calculating using pen and paper. She stresses that this is something she only thinks about now, pointing out that she has not considered this carefully. The same pattern is repeated when discussing the mathematical belief statements: she cannot explain why she has answered the way she has. But she underlines that she likes strict rules, because then she does not need to think so much: "Opinionising leads to confusion." Her identity as a learner might be described as following a peripheral trajectory, aligned at some points but more to be viewed as non-participating at other times.

PST5 does not know if she likes the thought of becoming a teacher of mathematics, and she has few ideas and thoughts about what to expect from teacher education. But, as she says, she hopes "it fits": If teacher education fits with her way of learning mathematics, then she is ready to learn. She does not demand that teacher education should offer something specific for her to fit in to, because she does not seem to know what it is that would help her fit in. Trying to describe her mode of identification through her own schooling, it is easy to get a picture of "just hanging in there". She wants to align, but is not willing to offer much. She seems insecure and passive, and we can perhaps conclude that she has a lot of identity work ahead of her in terms of directing her trajectory towards what kind of mathematics teacher she will become, if any.

Discussion

It is not surprising that a teacher educator in mathematics might want to see novice PSTs identifying as having high levels of both MSE and SETM (HH), since this might indicate that they have a solid *knowledge* and *understanding* about mathematics. The scatterplot in Figure 2 reveals very few PSTs with HH measures, only approximately 5%; this is based only on an estimation of where to draw the line between medium and high levels of MSE and SETM, but regardless of where the line is drawn, the number of HH PSTs is low. However, the analysis above might change this initial wish, thereby turning this low number into a positive: The exploration of patterns in the relationships between MSE and SETM, against the background of beliefs about mathematics and personal experience, suggests that developing identities as teachers of mathematics are more complex than high MSE and SETM alone can indicate.

Previous literature presented in the background section of this paper suggests a correlation between MSE and SETM, but few have investigated this correlation closely. Analysis of the interview data in terms of Wenger's focus on modes of identification adds historical context to a two-fold focus on those PSTs representing the correlation between MSE and SETM and those who do not. Furthermore, in applying Wenger's modes of identification, a third dimension emerges, capturing PSTs' levels of reflection. Teaching mathematics is not all about *knowing*, it is also about *making judgements* about knowing, that is, *reflection*. PSTs labelled as HH might give the impression of possessing a high degree of competency in mathematics and mathematics teaching, but they need to be able to apply judgement in their application. This emergent third dimension thus supplements the contribution made by Wenger's (2012) concept of modes of identification with Biesta's (2012) emphasis on the importance of developing the ability to make educationally wise judgements: the question is not so much whether teachers should be competent to do things (as measured in this case through the MSE and SETM elements of the instrument) but is more about their ability to make *judgements* about how and when to deploy those competences. Those PSTs demonstrating imagination and engagement as modes of identification can be seen as reflective and able to make judgements, while those who express modes of alignment can be seen as more unreflective and more unable to make the multidimensional judgements a teacher of mathematics needs to make.

These analyses suggest that even a PST who emerged as LL could be highly reflective, and thus have a more positive starting point for their developing identity as a prospective teacher than one who was rated HH. As indicated by Arvold (2005), PSTs interact with their teacher education programs through the lens of prior experience and the beliefs and values that go with that experience. PST1 (HH) follows what could be described as an 'exercise' (or rule-bound) teaching paradigm which fits her aligned learner approach to mathematics; she does not make judgements on how to use her competences as a prospective teacher. Her personal epistemology has similarities with the one described by De Corte et al. (2002). PST5 (ML) is on a peripheral trajectory trying to align as a teacher of mathematics whenever "it fits her". She does not reflect on her learning and appears unable to make judgements about what it takes to become and be a teacher of mathematics. She does not seem to see herself as a teacher, but identifies herself as a learner even after crossing the boundaries to the new landscape of practices in University College. It is interesting that PST1 and PST5, having respectively almost the highest and lowest SETM measures of the 191 PSTs, both emerge as 'not reflective'.

Turning to the remaining three interviewees, PST2 (MM) became lost in upper secondary school mathematics without reflecting on what went wrong. As noted by Brown (2012), these past experiences influence SETM. Despite this past experience, during his first weeks as a PST he has developed a curiosity and a mode of identification of imagination. In contrast, PST3 (LL), who also got lost in upper secondary, decided not to participate further on the basis of his reflections on the kind of teaching he was exposed to, but as a PST he has developed an identity of imagination. The same goes for PST4 (MH) who also demonstrates a mode of identification of imagination as she reflects in depth on the challenges she expects to meet when she has to teach mathematics in elementary school.

Locating PSTs' different modes of identification in this way enables us to uncover their levels of reflection. The three PSTs displaying a mode of identification of imagination have either a medium or low MSE score, suggesting some important aspects of their past experience with mathematics. It is important to minimize the focus on this last experience, as their level of reflection is a more positive starting point for their development as PSTs.

Conclusion

This study set out to investigate the connections between novice PSTs' perceptions of their own subject knowledge (as in MSE) and their self-efficacy as a potential teacher in mathematics (as in SETM), and to consider the implications for the identity work they need to do. While the findings repeat other research in finding a correlation, the mode of analysis and data collection employed enables us to see that regardless of whether or not they fit the correlation between MSE and SETM, PSTs also present a diverse range of identifications and trajectories, underlining some of the complexity we have to take into account as teacher educators in mathematics.

Previous research suggests that prior experience as a pupil relates to PSTs' trajectories within a new practice, and it is important to realise that each of the 191 PSTs in this study has a different story. By distinguishing between those who represent the correlation and those who do not, the analysis here reveals that the relationship between MSE and SETM alone does not paint a full picture of the range of PSTs and their nascent teacher identities. The inclusion of data on their previous experiences and their beliefs about mathematics teaching and learning enables a detailed investigation of the connections between PSTs' perceptions of their own subject knowledge and self-efficacy as a potential teacher in mathematics. A major emergent finding of this analysis has underlined the identity work they need to do in relation to *reflection*.

A new dimension of reflection is therefore needed to fine tune the picture. Their lengthy experience as observers of mathematics teaching can make it more difficult for them to imagine alternative approaches to teaching from those which they received in their own schooling (Prescott & Cavanagh, 2006), but in order to be reflective and to be able to make judgements, PSTs need to either imagine or engage in the new practices which they are entering. None of the five PSTs analysed in this paper showed engagement (in terms of a mode of identification) in their identity as a future teacher of mathematics. This raises an interesting question: What kind of identity work is needed in order to develop a mode of identification labelled as engagement? Further research is needed to show how PSTs can achieve engagement within a community of prospective teachers of mathematics and how this mode of identification and the ability to make educational judgements can ensure that they build SETM through teacher education. This study has shown that PSTs are a complex group and their perception of their own subject knowledge, their previous experience as learners and their self-efficacy as potential teachers in mathematics is important to take into consideration. We need to know more about this in order to provide a good teacher education.

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